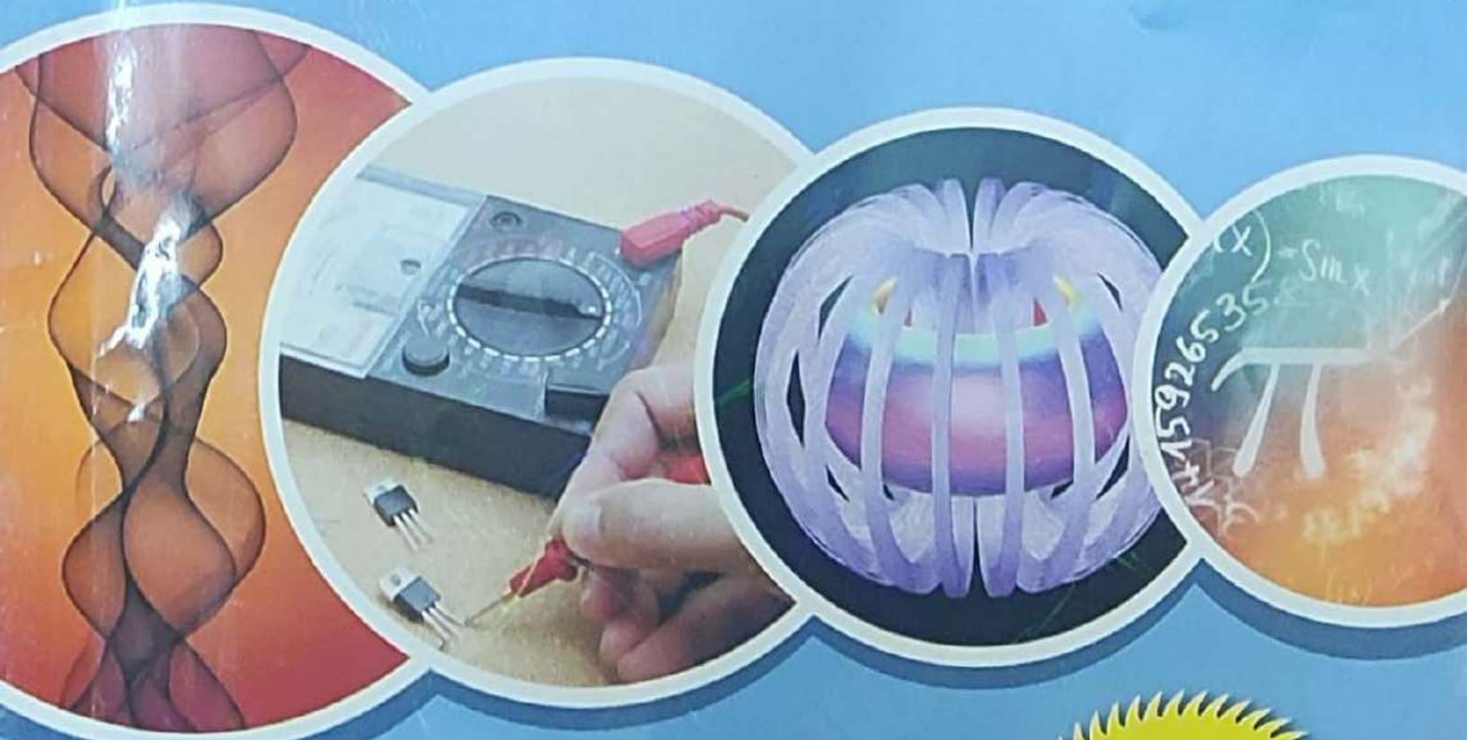
Semester - 3 Electrical & Electronics Engineering

Electromagnetic Field Theory



Session
2023-24
Odd Semester

- Topic-wise coverage of entire syllabus in Question-Answer form.
- Short Questions (2 Marks)

Includes solution of following AKTU Question Papers
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CONTENTS

BEE 301 : Electromagnetic Field Theory

1

UNIT

Coordinate Systems

CONTENTS

PART-1

Basics of Vectors : Addition, Subtraction and Multiplications.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.1. **What do you understand by scalar and vector quantity ? What is unit vector ?**

Answer

A. **Scalar quantity :** A quantity that has only magnitude and not direction is called, scalar quantity.

Example : Mass, time, density, temperature etc.

B. **Vector quantity :** A quantity that has both magnitude and direction is called vector quantity. A vector has starting point and end point.

Example : Force, displacement, velocity etc.

C. **Unit vector :** A unit vector is defined as a vector whose magnitude is unity. Unit vector in the direction of vector $\vec{A}$ is

$$\hat{a}_A = \frac{\vec{A}}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = \frac{\vec{A}}{|\vec{A}|}$$

Que 1.2. **Explain addition and subtraction of vectors.**

Answer

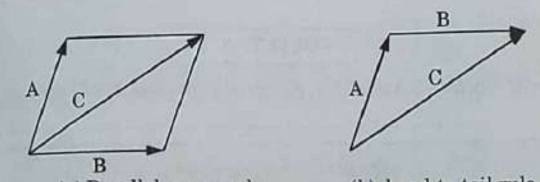
1. The vector $\vec{A}$ and $\vec{B}$ can be added together to give another vector $\vec{C}$;

$$\vec{C} = \vec{A} + \vec{B}$$

2. The vector addition is carried out component by component.

If $\vec{A} = (\vec{A}_x, \vec{A}_y, \vec{A}_z)$ and $\vec{B} = (\vec{B}_x, \vec{B}_y, \vec{B}_z)$,

then, $C = (A_x + B_x)\hat{a}_x + (A_y + B_y)\hat{a}_y + (A_z + B_z)\hat{a}_z$



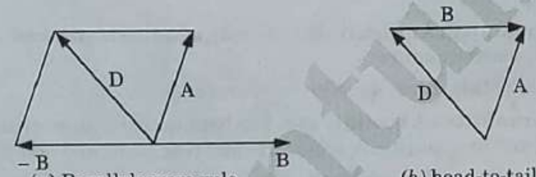
(*a*) Parallelogram rule (*b*) head-to-tail rule.

Fig. 1.2.1. Vector addition $\vec{C} = \vec{A} + \vec{B}$.

3. Vector subtraction is similarly carried out as,

$$\vec{D} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

$$= (A_x - B_x)\,\hat{a}_x + (A_y - B_y)\,\hat{a}_y + (A_z - B_z)\,\hat{a}_z$$



(*a*) Parallelogram rule (*b*) head-to-tail rule.

Fig. 1.2.2. Vector subtraction $\vec{D} = \vec{A} - \vec{B}$.

4. Graphically, vector addition and subtraction are obtained by either parallelogram rule or the head-to-tail rule in Fig. 1.2.1 and 1.2.2 respectively.

Que 1.3. **If $\vec{A} = 10\,\hat{a}_x - 4\,\hat{a}_y + 6\,\hat{a}_z$ and $\vec{B} = 2\,\hat{a}_x + \hat{a}_y$, find :**

i. The component of $\vec{A}$ along $\hat{a}_y$

ii. The magnitude of $3\vec{A} - \vec{B}$

iii. A unit vector along $\vec{A} + 2\vec{B}$.

Answer

Given, $\vec{A} = 10\hat{a}_x - 4\hat{a}_y + 6\hat{a}_z, \quad \vec{B} = 2\hat{a}_x + \hat{a}_y$

i. The component of $\vec{A}$ along $\hat{a}_y$ is

$$A_y = \vec{A}\,.\,\hat{a}_y = -4$$

ii. $$3\vec{A} - \vec{B} = 3(10\hat{a}_x - 4\hat{a}_y + 6\hat{a}_z) - (2\hat{a}_x + \hat{a}_y)$$

$$= 28\hat{a}_x - 13\hat{a}_y + 18\hat{a}_z$$

Magnitude of $|3\vec{A} - \vec{B}| = \sqrt{(28)^2 + (13)^2 + (18)^2} = 35.74$

iii. $$\vec{A} + 2\vec{B} = (10\hat{a}_x - 4\hat{a}_y + 6\hat{a}_z) + 2\,(2\hat{a}_x + \hat{a}_y)$$

$$= 14\hat{a}_x - 2\hat{a}_y + 6\hat{a}_z$$

Now, $|\vec{A} + 2\vec{B}| = \sqrt{(14)^2 + (2)^2 + (6)^2} = 15.36$

Unit vector of $\vec{A} + 2\vec{B} = \dfrac{\vec{A} + 2\vec{B}}{|\vec{A} + 2\vec{B}|} = \dfrac{14\hat{a}_x - 2\hat{a}_y + 6\hat{a}_z}{15.36}$

$$= 0.91\hat{a}_x - 0.13\,\hat{a}_y + 0.39\,\hat{a}_z$$

Que 1.4. **What is a dot product ? Also mention its properties and applications.**

Answer

A. Scalar (or Dot) product :

1. The scalar or dot product of the two vectors $\vec{A}$ and $\vec{B}$ is defined as the product of the magnitude of $\vec{A}$, the magnitude of $\vec{B}$ and the cosine of the smaller angle between them.

$$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta_{AB}$$

2. In dot product

$$\vec{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1$$

$$(\because\ \vec{i}\cdot\vec{i} = i\,i\cos\theta = 1 \text{ since } \theta = 0)$$

and $$\vec{i}\cdot\vec{j} = \vec{j}\cdot\vec{k} = \vec{k}\cdot\vec{i} = 0$$

$$(\because\ \vec{i}\cdot\vec{j} = i\,j\cos\theta = 0 \text{ since } \theta = 90°)$$

B. Properties of dot product :

1. If the two vectors are parallel to each other, *i.e.* $\theta = 0°$ then $\cos\theta_{AB} = 1$

then, $\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|$ for parallel vectors

2. If the two vectors are perpendicular to each other, *i.e.*, $\theta = 90°$ then $\cos\theta_{AB} = 0$ thus,

$$\vec{A}\cdot\vec{B} = 0 \text{ for perpendicular vector}$$

C. Applications of dot product :

1. To find the angle between the two vectors.

$$\theta = \cos^{-1}\left\{\frac{\vec{A}\cdot\vec{B}\cdot}{|\vec{A}|\,|\vec{B}|}\right\}$$

3. Physically, work done by a constant force $\vec{F}$ over a straight displacement $\vec{d}$ can be expressed as a dot product of two vectors.

$$W = \vec{F}\cdot\vec{d} = |\vec{F}|\,|\vec{d}|\cos\theta$$

Que 1.5. **What is a cross product ? Also mention its properties.**

Answer

A. Cross (or vector) product :

1. The cross or vector product is defined as the product of the magnitudes of $\vec{A}$ and $\vec{B}$ and the sine of the smaller angle between $\vec{A}$ and $\vec{B}$. But this product is a vector quantity and has a direction perpendicular to the plane containing the two vector $\vec{A}$ and $\vec{B}$.

$$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta_{AB}\,\hat{a}_N$$

where $\hat{a}_N$ = Unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B}$.

2. In cross product $\vec{i}\times\vec{i} = \vec{j}\times\vec{j} = \vec{k}\times\vec{k} = 0$

and $\vec{i}\times\vec{j} = \vec{k},\ \vec{j}\times\vec{k} = \vec{i},\ \vec{k}\times\vec{i} = \vec{j}$

Also $\vec{j}\times\vec{i} = \vec{k},\ \vec{k}\times\vec{j} = -\vec{i},\ \vec{i}\times\vec{k} = -\vec{j}$

B. Properties of cross product :

1. If cross product of two vectors is zero then those two vectors are parallel.

$\therefore\ \vec{A}\times\vec{B} = |\vec{A}|\,|\vec{B}|\sin\theta = 0$ (since $\theta = 0°$)

2. The cross product of a vector $\vec{A}$ with itself is zero as $\theta = 0°$

$\therefore\ \vec{A}\times\vec{A} = 0$

3. **Cross product in determinant form :** Consider the two vector in the cartesian system as,

$$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$

and $$\vec{B} = B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z$$

$$\vec{A}\times\vec{B} = \begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

Que 1.6. **If $\vec{A} = 3\hat{a}_r + 2\hat{a}_\theta - 6\hat{a}_\phi$ and $\vec{B} = 4\hat{a}_r + 3\hat{a}_\phi$. Determine :**

i. $\vec{A}\cdot\vec{B}$ **ii.** $|\vec{A}\times\vec{B}|$

Answer

Given, $\vec{A} = 3\hat{a}_r + 2\hat{a}_\theta - 6\hat{a}_\phi,\ \vec{B} = 4\hat{a}_r + 3\hat{a}_\phi$

i. $\vec{A}\cdot\vec{B} = (3\hat{a}_r + 2\hat{a}_\theta - 6\hat{a}_\phi)\cdot(4\hat{a}_r + 0\hat{a}_\theta + 3\hat{a}_\phi)$

$\vec{A}\cdot\vec{B} = 12 + 0 - 18 = -6$

ii. $$\vec{A}\times\vec{B} = \begin{vmatrix}\hat{a}_r & \hat{a}_\theta & \hat{a}_\phi\\ 3 & 2 & -6\\ 4 & 0 & 3\end{vmatrix}$$

$= \hat{a}_r[6] - \hat{a}_\theta[9 + 24] + \hat{a}_\phi[-8]$

$= 6\hat{a}_r - 33\hat{a}_\theta - 8\hat{a}_\phi$

$|\vec{A}\times\vec{B}| = \sqrt{6^2 + 33^2 + 8^2} = 34.48$

PART-2

Cartesian, Cylindrical, Spherical Transformation.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.7. **Write a short note on cartesian coordinate system.**

Answer

1. It is also known as rectangular coordinate system.
2. In rectangular coordinate system, three coordinate axis, *i.e.*, x, y and z are set up mutually at right angles to each other.
3. A point P in cartesian coordinate system is represented by P(x, y, z).

4. The ranges of the coordinate variables x, y and z are

$$-\infty < x < \infty, -\infty < y < \infty \text{ and } -\infty < z < \infty$$

5. Vector $\vec{A}$ in cartesian coordinate system can be written as (A_x, A_y, A_z) or

$A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$ where $\hat{a}_x, \hat{a}_y$ and $\hat{a}_z$ are unit vectors along the x, y and z directions respectively.

Que 1.8. **Give a brief description on cylindrical coordinate system.**

OR

Convert the cartesian coordinate system into cylindrical coordinate system.

Answer

A. Cylindrical coordinate system :

1. A point P (ρ, ϕ, z) in cylindrical coordinate system represents ρ the radius of the cylinder, ϕ the azimuthal angle and z is same as in cartesian system.
2. The ranges of variables ρ, ϕ and z are

$$0 \le \rho < \infty, 0 \le \phi < 2\pi \text{ and } -\infty < z < \infty$$

3. Vector $\vec{A}$ in cylindrical coordinate is written as (A_ρ, A_ϕ, A_z) or

$A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$ where $\hat{a}_\rho, \hat{a}_\phi$ and $\hat{a}_z$ are unit vectors in the ρ, ϕ and z directions respectively.

B. Relation between cartesian and cylindrical coordinate system :

$$\rho = \sqrt{x^2 + y^2}, \ \phi = \tan^{-1} y/x, \ z = z \quad ...(1.8.1)$$

1. Eq. (1.8.1) is used for transforming a point from cartesian to cylindrical coordinate.

$$x = \rho \cos \phi, \quad y = \rho \sin \phi, \quad z = z \quad ...(1.8.2)$$

2. Eq. (1.8.2) is used for transforming a point from cylindrical to cartesian.
3. In matrix form, we write the transformation of vector $\vec{A}$ from $(A_x, A_y, A_z) \to (A_\rho, A_\phi, A_z)$ is obtained as

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

4. The transformation $(A_\rho, A_\phi, A_z) \to (A_x, A_y, A_z)$ is obtained as

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Que 1.9. **Explain a point coordinate and all possible surfaces in vector form in cylindrical coordinate system.**

AKTU 2018-19, Marks 07

Answer

1. The cylindrical coordinate systems involve sets of three mutually orthogonal surfaces.
2. For the cylindrical coordinates system, the three surfaces are a cylinder and two planes as shown in Fig. 1.9.1.

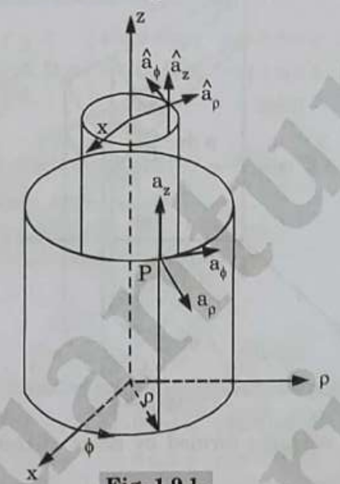


Fig. 1.9.1.

3. One of these planes is z (constant plane). The second plane contains the z-axis and makes an angle ϕ with reference plane. This plane is defined by ϕ which is a constant.
4. The cylindrical surface has the z-axis at its axis. Since the radial distance from the z-axis to point on the cylindrical surface is a constant, this surface is defined by which is equal to constant.
5. Only two of these coordinates (ρ and z) are distances and the third coordinates (ϕ) is an angle. The entire space is spanned by varying ρ from 0 to ∞, ϕ from 0 to 2π, and z from $-\infty$ to ∞.
6. The origin is given by $\rho = 0$, $\phi = 0$, and $z = 0$. But any other point in space is given by the intersection of three mutually orthogonal surfaces obtained by incrementing the coordinates by appropriate amounts.
7. The three vectors are represented as $\hat{a}_\rho, \hat{a}_\phi$ and $\hat{a}_z$.

i. $\hat{a}_\rho$ is directed radially outward normal to cylinder surface.

ii. $\hat{a}_\phi$ is perpendicular to ϕ = Constant plane and directed outward in the advance ϕ direction.

iii. $\hat{a}_z$ is perpendicular to z = constant plane.

8. To obtain expressions for the differential surfaces in the cylindrical coordinate system, we now consider two points $P(\rho, \phi, z)$ and $Q(\rho + d\rho, \phi + d\phi, z + dz)$, where Q is obtained by incrementing infinitesimally each coordinate from its value at P as shown in Fig. 1.9.2.

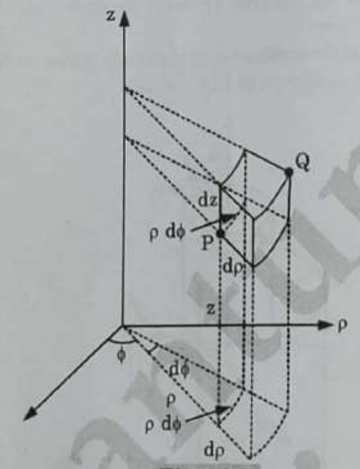


Fig. 1.9.2.

9. The differential surfaces formed by pairs of the differential length elements are

$$\pm dS\ \hat{a}_z = \pm (d\rho)(\rho\, d\phi)\,\hat{a}_z = \pm d\rho\ \hat{a}_\rho \times \rho\, d\phi\ \hat{a}_\phi$$

$$\pm dS\ \hat{a}_\rho = \pm (\rho\, d\phi)(d_z)\,\hat{a}_\rho = \pm \rho\, d\phi\ \hat{a}_\phi \times dz\ \hat{a}_z$$

$$\pm dS\ \hat{a}_\phi = \pm (dz)(d\rho)\,\hat{a}_\phi = \pm dz\ \hat{a}_z \times d\rho\ \hat{a}_\rho$$

Que 1.10. **Write a short note on spherical coordinate system.**

Answer

1. In a spherical coordinate system, a point P can be represented as $P(r, \theta, \phi)$.
2. Here, r represents radius of sphere, θ is the angle between z-axis and the position vector of P and ϕ is same as in cylindrical coordinate system.
3. The ranges of variables r, θ, ϕ are given by

$$0 \le r < \infty,\ 0 \le \theta \le \pi \text{ and } 0 \le \phi < 2\pi$$

4. The space variables (x, y, z) in cartesian coordinate can be related to variables (r, θ, ϕ) of spherical coordinate system as

$$r = \sqrt{x^2 + y^2 + z^2},\ \theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z},\ \phi = \tan^{-1} y/x$$

or $x = r \sin\theta \cos\phi,\ \ y = r\sin\theta\sin\phi,\ \ z = r\cos\theta$

5. In matrix form, the $(A_x, A_y, A_z) \rightarrow (A_r, A_\theta, A_\phi)$ vector transformation is performed according to

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

6. The inverse transformation $(A_r, A_\theta, A_\phi) \rightarrow (A_x, A_y, A_z)$ is obtained by

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

Que 1.11. **Given vector field $\vec{G} = 8 \sin\phi\, \hat{a}_r$ is in spherical coordinate, transform it into :**

i. Rectangular coordinate

ii. Cylindrical coordinate.

Answer

Given, $\vec{G} = 8\sin\phi\ \hat{a}_r$

i. Spherical coordinates to rectangular coordinates :

1. $$\begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} 8\sin\phi \\ 0 \\ 0 \end{bmatrix}$$

$$G_x = 8\sin\phi\sin\theta\cos\phi \quad \text{...(1.11.1)}$$

$$G_y = 8\sin\phi\sin\theta\sin\phi \quad \text{...(1.11.2)}$$

$$G_z = 8\sin\phi\cos\theta \quad \text{...(1.11.3)}$$

2. We know that $$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z}$$

and $$\phi = \tan^{-1}\frac{y}{x}$$

3. $$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\therefore \quad \sin\theta = \sqrt{1 - \cos^2\theta} = \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}}$$

$$\sin\phi = \frac{y}{r\sin\theta} = \frac{y}{\sqrt{x^2+y^2}}$$

$$\cos\phi = \frac{x}{r\sin\theta} = \frac{x}{\sqrt{x^2+y^2}}$$

4. Substituting all these values in eq. (1.11.1), (1.11.2) and (1.11.3), we get

$$G_x = \frac{8y}{\sqrt{x^2+y^2}}\,\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\,\frac{x}{\sqrt{x^2+y^2}}$$

$$= \frac{8xy}{\sqrt{x^2+y^2}\sqrt{x^2+y^2+z^2}}$$

$$G_y = \frac{8y}{\sqrt{x^2+y^2}}\,\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\,\frac{y}{\sqrt{x^2+y^2}}$$

$$= \frac{8y^2}{\sqrt{x^2+y^2}\sqrt{x^2+y^2+z^2}}$$

$$G_z = \frac{8yz}{\sqrt{x^2+y^2}\sqrt{x^2+y^2+z^2}}$$

Hence, $\vec{G} = G_x\,\hat{a}_x + G_y\,\hat{a}_y + G_z\,\hat{a}_z$

ii. Spherical to cylindrical coordinates :

1. $$\begin{bmatrix} G_\rho \\ G_\phi \\ G_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix}\begin{bmatrix} 8\sin\phi \\ 0 \\ 0 \end{bmatrix}$$

$$G_\rho = 8\sin\phi\sin\theta$$
$$G_\phi = 0$$
$$G_z = 8\sin\phi\cos\theta$$

2. We know that, $r = \sqrt{\rho^2+z^2}$

and $\theta = \tan^{-1}\dfrac{\rho}{z}$

3. Thus, $$\sin\theta = \frac{\rho}{\sqrt{\rho^2+z^2}}$$

$$\cos\theta = \frac{z}{\sqrt{\rho^2+z^2}}$$

4. $$G_\rho = \frac{8\rho\sin\phi}{\sqrt{\rho^2+z^2}}$$

$$G_z = \frac{8z\sin\phi}{\sqrt{\rho^2+z^2}}$$

5. Hence, $$\vec{G} = \frac{8\rho\sin\phi}{\sqrt{\rho^2+z^2}}\,\hat{a}_\rho + \frac{8z\sin\phi}{\sqrt{\rho^2+z^2}}\,\hat{a}_z$$

Que 1.12. Transform the $A = r\,a_r$ into cartesian and cylindrical coordinate system. **AKTU 2018-19, Marks 07**

Answer

The procedure is same as Q. 1.11, Page 1–10B, Unit-1.

A. Spherical coordinate to cartesian coordinate system,

$$\overline{A} = \frac{ax}{\sqrt{x^2+y^2+z^2}}\,\hat{a}_x + \frac{ay}{\sqrt{x^2+y^2+z^2}}\,\hat{a}_y + \frac{az}{\sqrt{x^2+y^2+z^2}}\,\hat{a}_z$$

B. Spherical coordinate to cylindrical coordinate,

$$\overline{A} = \frac{a\rho}{\sqrt{\rho^2+z^2}}\,\hat{a}_\rho + \frac{az}{\sqrt{\rho^2+z^2}}\,\hat{a}_z$$

Que 1.13. Convert a point P (4, – 3, 6) and a vector $R = z\,a_x + y\,a_z$ into cylindrical coordinate systems. **AKTU 2019-20, Marks 10**

Answer

1. At point P, $x = 4$, $y = -3$, $z = 6$

$$\rho = \sqrt{x^2+y^2} = \sqrt{(4)^2+(-3)^2} = \sqrt{16+9} = 5$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-3}{4}\right)$$
$$= \tan^{-1}(-0.75)$$
$$= -36.86$$

Thus $P(4, -3, 6) = P(5, -36.86, 6)$

2. For vector R, $R_x = z$, $R_y = 0$ $R_z = y$. Hence, in the cylindrical system

$$\begin{bmatrix} R_\rho \\ R_\phi \\ R_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} z \\ o \\ y \end{bmatrix}$$

$$R_\rho = z\cos\phi \quad ...(1.13.1)$$
$$R_\phi = -z\sin\phi \quad ...(1.13.2)$$
$$R_z = y \quad ...(1.13.3)$$

3. But $x = \rho\cos\phi$, $y = \rho\sin\phi$, put the value of y in eq. (1.13.3), we get

$$R = (R_\rho, R_\phi, R_z)$$
$$= [z\cos\phi]\,a_\rho + [-z\sin\phi]\,a_\phi + [\rho\sin\phi]\,a_z$$

4. At point P

$$\rho = 5 \qquad \tan\phi = \frac{-3}{4}$$

$$\cos\phi = \frac{x}{\rho} = \frac{4}{5}$$

$$\sin\phi = \frac{y}{\rho} = \frac{-3}{5}$$

$$R = \left[6\times\frac{4}{5}\right]a_\rho + \left[-6\times\frac{-3}{5}\right]a_\phi + \left[5\times\frac{-3}{5}\right]a_z$$

$$= \left[\frac{24}{5}\right]a_\rho + \left[\frac{18}{5}\right]a_\phi + [-3]\,a_z$$

5. $$R = 4.8\,a_\rho + 3.6\,a_\phi - 3\,a_z$$

Que 1.14. **Investigate the values of X, Y and Z. If $\vec{A} = (2\hat{a}_x + 4\hat{a}_y + 5\hat{a}_z)$ is transformed as $\vec{A} = (X\hat{a}_r + Y\hat{a}_\theta + Z\hat{a}_\phi)$**

AKTU 2021-22, Marks 10

Answer

1. Vector $\vec{A}$ expressed in the cartesian system as,

$$\vec{A} = 2\hat{a}_x + 4\hat{a}_y + 5\hat{a}_z$$

2. The component of $\vec{A}$ in $\hat{a}_r$ direction is given by,

$$A_r = \vec{A}.\hat{a}_r$$
$$= (2\hat{a}_x + 4\hat{a}_y + 5\hat{a}_z).\hat{a}_r$$
$$= 2\hat{a}_x.\hat{a}_r + 4\hat{a}_y.\hat{a}_r + 5\hat{a}_z.\hat{a}_r$$
$$= 2\sin\theta.\cos\phi + 4.\sin\theta.\sin\phi + 5\cos\theta$$

3. The component of $\vec{A}$ in $\hat{a}_\theta$ direction is given by,

$$A_\theta = \vec{A}.\hat{a}_\theta$$
$$= (2\hat{a}_x + 4\hat{a}_y + 5\hat{a}_z).\hat{a}_\theta$$
$$= 2\hat{a}_x.\hat{a}_\theta + 4\hat{a}_y.\hat{a}_\theta + 5\hat{a}_z.\hat{a}_\theta$$
$$= 2\cos\theta.\cos\phi + 4\cos\theta\sin\theta - 5\sin\theta$$

4. The component of $\vec{A}$ in $\hat{a}_\phi$ direction is given by,

$$A_\phi = \vec{A}.\hat{a}_\phi$$
$$= (2\hat{a}_x + 4\hat{a}_y + 5\hat{a}_z).\hat{a}_\phi$$

$$= 2\hat{a}_x.\hat{a}_\phi + 4\hat{a}_y.\hat{a}_\phi + 5\hat{a}_z.\hat{a}_\phi$$
$$A_\phi = -2\sin\phi + 4\cos\phi$$

5. Collecting these results, we have

$$A = A_r + A_\theta + A_\phi$$
$$= (2.\sin\theta.\cos\phi + 4\sin\theta.\sin\phi + 5\cos\theta).\hat{a}_r$$
$$+ (2\cos\theta.\cos\phi + 4\cos\theta.\sin\theta - 5\sin\theta).\hat{a}_\theta$$
$$+ (-2\sin\phi + 4\cos\phi).\hat{a}_\phi \quad ...(1.14.1)$$

Given, $$\vec{A} = X\hat{a}_r + Y\hat{a}_\theta + Z\hat{a}_\phi \quad ...(1.14.2)$$

6. Comparing eq. (1.14.1) and (1.14.2), we get

$$X = 2\sin\theta.\cos\phi + 4\sin\theta.\sin\phi + 5\cos\theta$$
$$Y = 2\cos\theta.\cos\phi + 4\cos\theta.\sin\theta - 5\sin\theta$$
$$Z = -2\sin\phi + 4\cos\phi$$

PART-3

Vector Calculus : Differential Length, Area and Volume.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.15. **Define the following quantities :**

i. Differential length **ii. Differential area**
iii. Differential volume.

Answer

i. Differential length :

1. It is the smallest part of length l. It is denoted by 'dl'.
2. In cartesian coordinate system, length $\vec{dl}$ is given by

$$\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

3. In cylindrical coordinate system,

$$\vec{dl} = d\rho\,\hat{a}_\rho + \rho\,d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$$

4. In spherical coordinate system,

$$\vec{dl} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$$

ii. Differential area :

1. It is the differential part of surface area S. It is denoted by '$\vec{dS}$'.

2. In cartesian coordinate system, surface $\vec{dS}$ is given by

$$\vec{dS} = dy\,dz\,\hat{a}_x = dz\,dx\,\hat{a}_y = dx\,dy\,\hat{a}_z$$

3. In cylindrical coordinate system,

$$\vec{dS} = \rho\,d\phi\,dz\,\hat{a}_\rho = d\rho\,dz\,\hat{a}_\phi = \rho\,d\rho\,d\phi\,\hat{a}_z$$

4. In spherical coordinate system

$$\vec{dS} = r^2\sin\theta\,d\theta\,d\phi\;\hat{a}_r = r\sin\theta\,dr\,d\phi\;\hat{a}_\theta = r\,dr\,d\theta\;\hat{a}_\phi$$

iii. Differential volume :

1. It is the differential part of volume v. It is denoted by dv.
2. In cartesian coordinate system differential volume is given by
$$dv = dx\,dy\,dz$$
3. In cylindrical coordinate system,
$$dv = \rho\,d\rho\,d\phi\,dz$$
4. In spherical coordinate system,
$$dv = r^2\sin\theta\,dr\,d\theta\,d\phi$$

PART-4

Line, Surface and Volume Integrals.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.16. Define following quantities :

i. Line integral
ii. Surface integral
ii. Volume integral.

OR

Explain all possible forms of surface vector, line vector and volume integral in spherical system. **AKTU 2019-20, Marks 10**

Answer

i. Line integral :

1. Line integral is defined as any integral which is to be evaluated along a curve.
2. The line integral $\int_L \vec{A}\cdot\vec{dl}$ is the integral of the tangential component of $\vec{A}$ along curve $\vec{L}$.

3. The line integral of $\vec{A}$ around L is defined as :

$$\int_L \vec{A}\cdot\vec{dl} = \int_p^r |\vec{A}|\cos\theta\,dl \qquad ...(1.16.1)$$

where, $\vec{dl}$ = elemental length.

4. If the path of integration is a closed curve such as p-q-r-s-p in Fig. 1.16.1, eq. (1.16.1) becomes a closed contour integral.

$$\oint_L \vec{A}\cdot\vec{dl} = \text{Circular integral} \qquad ...(1.16.2)$$

5. If $\oint \vec{A}\cdot\vec{dl} = 0$, then the vector field is said to be conservative.

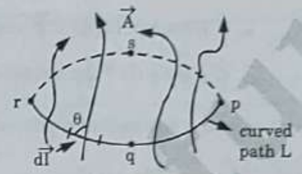


Fig. 1.16.1.

ii. Surface integral :

1. The surface integral or the flux of vector field $\vec{A}$ through surface S is defined as

$$\psi = \int_S \vec{A}\cdot\vec{dS} = \int_S |\vec{A}|\,dS\cos\theta = \int_S \vec{A}\cdot\hat{a}_n\,dS$$

where, $\hat{a}_n$ = Unit vector normal to the surface S.

2. It becomes a double integration while solving the problems.
3. For the closed surface,

$$\psi = \oint_S \vec{A}\cdot\vec{dS}$$

4. This represents the net outward flux of vector field $\vec{A}$ from surface S.

iii. Volume integral :

1. Volume integral of the scalar ρ_v over the volume is defined as,

$$Q = \int_v \rho_v\,dv$$

2. Since, volume is a three dimensional quantity hence it becomes a triple integral.

Que 1.17. Determine close path line integral of $A = \rho\cos\phi\;a_\rho + \sin\phi\;a_\phi$ around a circle which is placed in x-y plane with origin as a center. **AKTU 2018-19, Marks 07**

Answer

Given : $A = \rho \cos\phi\, \hat{\rho} + \sin\phi\, \hat{\phi}$

To Find : Closed path line integral $= \oint \vec{A}\,.\,d\vec{l}$.

1. Here $$d\vec{l} = d\rho\,\hat{\rho} + \rho d\phi\,\hat{\phi} + dz\,\hat{z}$$
2. $$\oint \vec{A}\,.\,d\vec{l} = \oint (\rho\cos\phi\,\hat{\rho} + \sin\phi\,\hat{\phi})\cdot(d\rho\hat{\rho} + \rho d\phi\,\hat{\phi} + dz\,\hat{z})$$
$$= \oint \rho\cos\phi\,\hat{\rho}\,.\,(d\rho\hat{\rho} + \rho d\phi\,\hat{\phi} + dz\,\hat{z}) + \oint \sin\phi\,\hat{\phi}\,.\,(d\rho\,\hat{\rho} + \rho d\phi\,\hat{\phi} + dz\,\hat{z})$$
$$= \oint \rho\cos\phi\,\hat{\rho}\,.\,(d\rho\hat{\rho}) + \oint \sin\phi\,\hat{\phi}\,.(\rho d\phi\,\hat{\phi})$$
$$= \oint \rho\cos\phi\, d\rho + \oint \rho\sin\phi\, d\phi$$
3. x-y plane :

Fig. 1.18.1.

4. Around circle, $d\rho = 0$

$$\therefore \quad \oint \vec{A}\,.\,d\vec{l} = \int_0^{2\pi} \rho\sin\phi\, d\phi = \rho[-\cos\phi]_0^{2\pi} = 0$$

PART-5

Del Operator, Gradient.

Questions-Answers
Long Answer Type and Medium Answer Type Questions

Que 1.18. **What is 'Del operator' ?**

Answer

1. It is also known as vector differential operator. It can also be named as "nabla" and is represented by symbol ∇.
2. The del operator is useful in defining :

i. The gradient of scalar V, written as ∇V

ii. The divergence of vector $\vec{A}$, written as $\nabla\cdot\vec{A}$

iii. The curl of a vector $\vec{A}$, written as $\nabla\times\vec{A}$

iv. The Laplacian of a scalar V, written as $\nabla^2 V$.

3. In cartesian coordinate system,
$$\boxed{\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z}$$
4. In cylindrical coordinate system,
$$\boxed{\nabla = \frac{\partial}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}\hat{a}_\phi + \frac{\partial}{\partial z}\hat{a}_z}$$
5. In spherical coordinate system,
$$\boxed{\nabla = \frac{\partial}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\hat{a}_\phi}$$

Que 1.19. **Describe the gradient of a scalar field.**

AKTU 2017-18, Marks 3.5

OR

Write the properties of the gradient of a scalar.

Answer

A. Gradient of scalar field :

1. The gradient of a scalar field M is a vector that represents both the magnitude and the direction of the maximum space rate of increase of M.
2. The operation of the ∇(del) operator on a scalar function is called gradient of a scalar.

So, $$\text{grad}\, M = \nabla M = \left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right) M$$
$$\nabla M = \frac{\partial M}{\partial x}\hat{a}_x + \frac{\partial M}{\partial y}\hat{a}_y + \frac{\partial M}{\partial z}\hat{a}_z$$

3. In cylindrical coordinate system,
$$\nabla M = \frac{\partial M}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial M}{\partial \phi}\hat{a}_\phi + \frac{\partial M}{\partial z}\hat{a}_z$$
4. In spherical coordinate system,
$$\nabla M = \frac{\partial M}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial M}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial M}{\partial \phi}\hat{a}_\phi$$

B. Properties of gradient of scalar :

1. The gradient ∇M gives the maximum rate of change of M per unit distance.

2. The gradient ∇M always indicates the direction of the maximum rate of change of M.
3. The gradient ∇M at any point is perpendicular to the constant M surface, which passes through the point.
4. The directional derivative of M along the unit vector $\hat{a}$ is $\nabla M.\hat{a}$, which is projection of ∇M in the direction of unit vector $\hat{a}$.

Que 1.20. **Find the gradient of the following scalar field :**

i. $V = e^{-z} \sin 2x \cosh y$
ii. $U = \rho^2 z \cos 2\phi$

Answer

i. Given, $V = e^{-z} \sin 2x \cosh y$

$$\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

$$= 2e^{-z} \cos 2x \cosh y\, \hat{a}_x + e^{-z} \sin 2x \sinh y\, \hat{a}_y - e^{-z} \sin 2x \cosh y\, \hat{a}_z$$

ii. Given, $V = \rho^2 z \cos 2\phi$

$$\nabla U = \frac{\partial U}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial U}{\partial \phi}\hat{a}_\phi + \frac{\partial U}{\partial z}\hat{a}_z$$

$$= 2\rho z \cos 2\phi\, \hat{a}_\rho - 2\rho z \sin 2\phi\, \hat{a}_\phi + \rho^2 \cos 2\phi\, \hat{a}_z$$

PART-6

Divergence of a Vector, Divergence Theorem.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.21. **Explain the divergence of a vector ? Also write the physical significance of divergence.**

Answer

A. Divergence of vector :

1. The divergence of $\vec{F}$ at a given point P is the outflow of flux per unit volume as the volume shrinks about P.

2. The divergence of $\vec{F}$ is defined as

$$\text{div } \vec{F} = \nabla.\vec{F} = \lim_{\Delta v \to 0} \frac{\oint_S \vec{F}\cdot d\vec{S}}{\Delta v}$$

3. Divergence of $\vec{F}$ in cartesian system is given by

$$\nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

4. Divergence in cylindrical coordinate is given by

$$\nabla \cdot \vec{F} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho F_\rho) + \frac{1}{\rho}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$$

5. Divergence in spherical coordinate is given by

$$\nabla \cdot \vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$$

6. $\nabla \cdot \vec{F} = 0$ for $\vec{F}$ to be solenoidal.

B. **Physical significance of divergence :** "Divergence of a vector function $\vec{F}$ at each point gives the rate per unit volume at which the physical entity is issuing from that point".

Que 1.22. **Prove that the total outward flux of a vector field $\vec{A}$ through the closed surfaces S is the same as the volume integral of the divergence of $\vec{A}$.**

OR

State and explain the divergence theorem.

Answer

A. Divergence theorem :

1. Gauss's divergence theorem states that "The volume integral of the divergence of a vector field $\vec{A}$ taken over any volume v is equal to the surface integral of $\vec{A}$ taken over the closed surface $\vec{S}$ surrounding the volume v."
2. Mathematically, Gauss's divergence theorem for any vector field $\vec{A}$ can be written as,

$$\iiint_v (\nabla \cdot \vec{A})\, dv = \iint_S \vec{A} \cdot d\vec{S} \quad \text{...(1.22.1)}$$

where, $d\vec{S}$ = An element of area on surface S

B. Proof :

1. Taking L.H.S. of eq. (1.22.1),

$$\iiint_v (\nabla \cdot \vec{A})\, dv = \iiint_v \left\{ \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right\} dx\, dy\, dz$$

$$= \iiint_v \left(\frac{\partial A_x}{\partial x} dx\, dy\, dz \right) + \iiint_v \left(\frac{\partial A_y}{\partial y} dx\, dy\, dz \right)$$

$$+ \iiint_v \left(\frac{\partial A_z}{\partial z} dx\, dy\, dz \right) \quad ...(1.22.2)$$

2. Take the first integral part of eq. (1.22.2),

i.e,
$$\iiint_v \left(\frac{\partial A_x}{\partial x} dx\, dy\, dz \right) \quad ...(1.22.3)$$

3. Consider the Fig. 1.22.1 in which a strip of cross-section which extends from Q_1 to Q_2.

$$\therefore \quad \int_{Q_1 = x_1}^{Q_2 = x_2} \frac{\partial A_x}{\partial x} dx = [A_x]_{x_1}^{x_2}$$

or
$$\int_{x_1}^{x_2} \frac{\partial A_x}{\partial x} dx = (A_{x2} - A_{x1}) \quad ...(1.22.4)$$

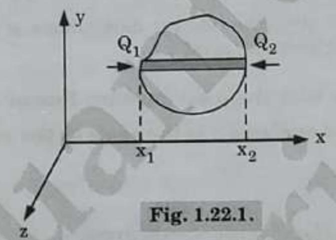


Fig. 1.22.1.

4. So,
$$\iiint_v \frac{\partial A_x}{\partial x} dx\, dy\, dz = \iint_S \left[\int \frac{\partial A_x}{\partial x} dx \right] dy\, dz$$

$$= \iint_S [A_{x_2} - A_{x_1}]\, dy\, dz$$

or
$$\iiint_v \frac{\partial A_x}{\partial x} dx\, dy\, dz = \iint_S A_x\, dS_x \quad ...(1.22.5)$$

Similarly,
$$\iiint_v \frac{\partial A_y}{\partial y} dx\, dy\, dz = \iint_S A_y\, dS_y \quad ...(1.22.6)$$

and
$$\iiint_v \frac{\partial A_z}{\partial z} dx\, dy\, dz = \iint_S A_z\, dS_z \quad ...(1.22.7)$$

5. On adding eq. (1.22.5), (1.22.6) and (1.22.7),

$$\iiint_v \frac{\partial A_x}{\partial x} dx\, dy\, dz + \iiint_v \frac{\partial A_y}{\partial y} dx\, dy\, dz + \iiint_v \frac{\partial A_z}{\partial z} dx\, dy\, dz$$

$$= \iint_S [A_x\, dS_x + A_y\, dS_y + A_z\, dS_z] = \iint_S [\vec{A} \cdot d\vec{S}]$$

or,
$$\iiint_v (\nabla \cdot \vec{A})\, dv = \iint_S \vec{A} \cdot d\vec{S} \quad ...(1.22.8)$$

6. Eq. (1.22.8) shows that the total outward flux of a vector field $\vec{A}$ through the closed surface S is same as volume integral of the divergence of $\vec{A}$. Hence, divergence theorem is proved.

Que 1.23. **Find the divergence of a vector**

$$A = 8x^2\, \hat{i}_x + 5\, x^2 y^2\, \hat{i}_y + xyz^3\, \hat{i}_z$$

and del (∇) of a scalar function x^2yz. **AKTU 2017-18, Marks 3.5**

Answer

1. Divergence of a vector A is given by,

$$\nabla \cdot \vec{A} = \frac{\partial}{\partial x} 8x^2 + \frac{\partial}{\partial y} 5x^2 y + \frac{\partial}{\partial z} xyz^3 = 16x + 5x^2 + 3xy\, z^2$$

2. Del (∇) of a scalar function x^2yz,

$$\nabla \cdot B = \frac{\partial}{\partial x} x^2 yz\, \hat{i}_x + \frac{\partial}{\partial y} x^2 yz\, \hat{i}_y + \frac{\partial}{\partial z} x^2 yz\, \hat{i}_z \quad (B = x^2 yz)$$

$$\nabla \cdot B = 2xyz\, \hat{i}_x + x^2 z\, \hat{i}_y + x^2 y\, \hat{i}_z$$

Que 1.24. **Given that $\vec{A} = \left(\frac{5r^2}{4} \right) \hat{a}_r$ is in spherical coordinates, solve both sides of the divergence theorem for the volume enclosed by $r = 4$ m, and $\theta = \frac{\pi}{4}$ shown in below Fig. 1.24.1.**

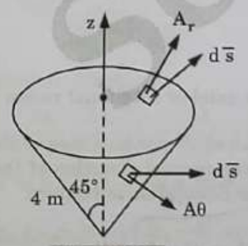


Fig. 1.24.1.

AKTU 2021-22, Marks 10

Answer

We know that

$$\oint A.ds = \oint (\nabla . A)dv$$

where, A has only a radial component. $A.ds$ has a non-zero value on the surface $r = 4$ m.

$$\oint A.ds = \int_0^{2\pi}\int_0^{\pi/4} \frac{5(4)^2}{4}\hat{a}_r .(4)^2 \sin\theta d\theta d\phi \hat{a}_r$$

$$= 589.1 \text{ C}$$

Divergence theorem gives

$$\nabla.A = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{5r^2}{4}\right) = 5r \text{ and}$$

$$\int(\nabla.A)dv = \int_0^{2\pi}\int_0^{\pi/4}\int_0^{4}(5r)r^2 \sin\theta dr d\phi = 589.1 \text{ C}$$

PART-7

Curl of a Vector.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.25. **What do you mean by curl of a vector ? Also write the properties and physical interpretation of curl.**

Answer

A. Curl of a vector :

1. The curl of $\vec{A}$ is an axial or rotational vector whose magnitude is the maximum circulation of $\vec{A}$ per unit area as the area tends to zero and whose direction is the normal direction of the area when the area is oriented to make the circulation maximum.

i.e., $$\boxed{\text{curl } \vec{A} = \nabla \times \vec{A}} = \left(\lim_{\Delta S \to 0} \oint_L \frac{\vec{A}\cdot d\vec{l}}{\Delta S}\right)_{max} \cdot \hat{a}_n$$

where the area ΔS is bounded by the curve L and $\hat{a}_n$ is the unit vector normal to the surface ΔS.

2. In cartesian coordinate system, the curl of $\vec{A}$ is

$$\nabla \times \vec{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\hat{a}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\hat{a}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\hat{a}_z$$

3. In cylindrical coordinates,

$$\nabla \times \vec{A} = \left[\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right]\hat{a}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right]\hat{a}_\phi + \frac{1}{\rho}\left[\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right]\hat{a}_z$$

4. In spherical coordinates,

$$\nabla \times \vec{A} = \frac{1}{r\sin\theta}\left[\frac{\partial(A_\phi \sin\theta)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right]\hat{a}_r$$

$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial(rA_\phi)}{\partial r}\right]\hat{a}_\theta + \frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]\hat{a}_\phi$$

B. Properties of the curl :

1. $\nabla \times (\vec{A} + \vec{B}) = (\nabla \times \vec{A}) + (\nabla \times \vec{B})$
2. $\nabla \times (\vec{A} \times \vec{B}) = \vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A}) + (\vec{B}\cdot\nabla)\vec{A} - (\vec{A}\cdot\nabla)\vec{B}$
3. $\nabla \times \nabla V = 0$, *i.e.*, the curl of the gradient of a scalar field vanished.
4. $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot A) - \nabla^2 A$

C. Physical interpretation of curl : The physical interpretation of a curl of a vector field is that the curl provides the maximum value of the circulation of the field per unit area and indicates the direction along which this maximum value occurs.

Que 1.26. **Given a vector function :**

$$\vec{A} = (3x + c_1 z)\,\hat{a}_x + (c_2 x - 5z)\,\hat{a}_y + (4x - c_3 y + c_4 z)\,\hat{a}_z.$$

Calculate c_1, c_2, c_3, c_4 if $\vec{A}$ is irrotational and solenoidal.

Answer

1. For $\vec{A}$ to be irrotational, $\nabla \times \vec{A} = 0$

$$\nabla \times \vec{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\hat{a}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\hat{a}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\hat{a}_z$$

$$A_x = 3x + c_1 z,\ A_y = c_2 x - 5z,\ A_z = 4x - c_3 y + c_4 z$$

$$\nabla \times \vec{A} = [-c_3 + 5]\ \hat{a}_x + [c_1 - 4]\ \hat{a}_y + [c_2 - 0]\ \hat{a}_z = 0$$

$$c_3 = 5, c_1 = 4, c_2 = 0$$

2. For $\vec{A}$ to be solenoidal, $\nabla \cdot \vec{A} = 0$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = 0$$

$$3 + 0 + c_4 = 0 \Rightarrow c_4 = -3$$

PART-8

Stoke's Theorem.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.27. **Explain and prove stroke's theorem.**

AKTU 2019-20, Marks 10

Answer

A. Stoke's theorem :

1. It states that the circulation of vector field $\vec{A}$ around a closed path L is equal to the surface integral of the curl of $\vec{A}$ over the open surface S bounded by L provided $\vec{A}$ and $\nabla \times \vec{A}$ are continuous on S.
2. This theorem is only applicable when $\vec{A}$ and $\nabla \times \vec{A}$ are continuous on the surface S. Mathematically, it is expressed as,

$$\oint_L \vec{A} \cdot d\vec{l} = \int_S (\nabla \times \vec{A}) \cdot d\vec{S}$$

B. Proof of Stoke's theorem :

1. Assume a surface S which is divided into a number of incremental surfaces ΔS as shown in Fig. 1.27.1.

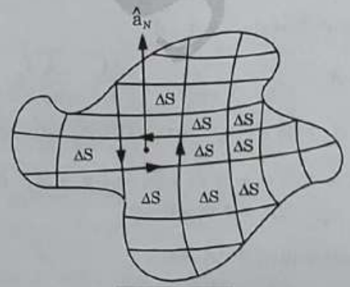


Fig. 1.27.1.

2. From the definition of curl,

$$(\nabla \times \vec{A})_N = \frac{\oint \vec{A} \cdot d\vec{l}_{\Delta S}}{\Delta S} \quad ...(1.27.1)$$

$N \rightarrow$ normal to ΔS.

$dl_{\Delta S} \rightarrow$ perimeter of surface ΔS.

3. According to right hand rule,

$$(\nabla \times \vec{A})_N = (\nabla \times \vec{A}) \cdot \hat{a}_N$$

Put this value in eq. (1.27.1),

$$\oint \vec{A} \cdot d\vec{l}_{\Delta S} = (\nabla \times \vec{A}) \cdot \hat{a}_N \, \Delta S$$

$$\oint \vec{A} \cdot d\vec{l}_{\Delta S} = (\nabla \times \vec{A}) \cdot \Delta \vec{S} \quad ...(1.27.2)$$

4. For obtaining the total curl, add the closed line integrals for each ΔS. From Fig. 1.27.2, it can be observed that there is a common boundary between the two incremental surfaces.
5. The line integral is getting cancelled as the boundary is getting traced in two opposite directions. All interior boundaries are cancelled; only outside boundary cancellation does not exist.

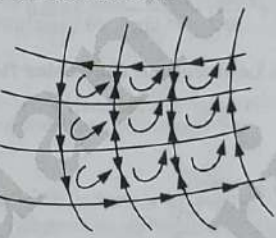


Fig. 1.27.2.

6. Hence, summation of all closed line integrals for each and every ΔS ends up in a single closed line integral to be obtained for the outer boundary of the total surface S. So, eq. (1.27.2) becomes,

$$\oint_L \vec{A} \cdot d\vec{l} = \int_S (\nabla \times \vec{A}) \cdot d\vec{S}$$

PART-9

Laplacian of a Scalar.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.28. **What do you mean by "Laplacian of scalar" ?**

Answer

1. The composite operator of 'divergence of a vector' and 'gradient of scalar' is called Laplacian of a scalar.
2. If A is a scalar field, then Laplacian of scalar A is denoted as $\nabla^2 A$.
3. It is defined as the divergence of the gradient of A. ∇^2 is called Laplacian operator.
4. In cartesian coordinate,

$$\nabla^2 A = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2}$$

5. In cylindrical coordinate,

$$\nabla^2 A = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial A}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 A}{\partial \phi^2}\right) + \frac{\partial^2 A}{\partial z^2}$$

6. In spherical coordinate,

$$\nabla^2 A = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial A}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial A}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 A}{\partial \phi^2}$$

Que 1.29. **Find the Laplacian of the scalar fields :**

$$V = e^{-z} \sin 2x \cosh y$$

Answer

Given, $V = e^{-z} \sin 2x \cosh y$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}(e^{-z} \sin 2x \cosh y)\right) + \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}(e^{-z} \sin 2x \cosh y)\right) + \frac{\partial}{\partial z}\left(\frac{\partial}{\partial z}(e^{-z} \sin 2x \cosh y)\right)$$

$$= \frac{\partial}{\partial x}(2e^{-z} \cos 2x \cosh y) + \frac{\partial}{\partial y}(e^{-z} \sin 2x \sinh y) + \frac{\partial}{\partial z}(-e^{-z} \sin 2x \cosh y)$$

$$= -4\, e^{-z} \sin 2x \cosh y + e^{-z} \sin 2x \cosh y + e^{-z} \sin 2x \cosh y$$

$$= -2\, e^{-z} \sin 2x \cosh y$$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. What is a dot product ? Also mention its properties and applications.

Ans. Refer Q. 1.4.

Q. 2. What is a cross product ? Also mention its properties.

Ans. Refer Q. 1.5.

Q. 3. Give a brief description on cylindrical coordinate system.

Ans. Refer Q. 1.8.

Q. 4. Transform the $A = r\, a_r$ into cartesian and cylindrical coordinate system.

Ans. Refer Q. 1.12.

Q. 5. Convert a point P (4, – 3, 6) and a vector $R = z\, a_x + y\, a_z$ into cylindrical coordinate systems.

Ans. Refer Q. 1.13.

Q. 6. Explain all possible forms of surface vector, line vector and volume integral in spherical system.

Ans. Refer Q. 1.16.

Q. 7. Describe the gradient of a scalar field.

Ans. Refer Q. 1.19.

Q. 8. State and explain the divergence theorem.

Ans. Refer Q. 1.22.

Q. 9. Find the divergence of a vector

$$A = 8x^2\, \hat{i}_x + 5\, x^2 y^2\, \hat{i}_y + xyz^3\, \hat{i}_z$$

and del ∇ of a scalar function x^2yz.

Ans. Refer Q. 1.23.

Q. 10. Explain and prove stroke's theorem.

Ans. Refer Q. 1.27.

☺☺☺

2

UNIT

Electrostatics

CONTENTS

PART-1

Coulombs Law and Field Intensity, Electric Field Due to Charge Distribution.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.1. **State and explain the Coulomb's law.**

Answer

1. Coulomb's law states that the force F between the two point charges Q_1 and Q_2 is directly proportional to the product of two charges and is inversely proportional to the square of the distance R between them. Mathematically,

$$\vec{F} = \frac{k\,Q_1 Q_2}{R^2}$$

2. The constant of proportionality k is written as

$$k = \frac{1}{4\pi \epsilon_0}$$

where, ϵ_0 = Permittivity of free space

$$\epsilon_0 = 8.854 \times 10^{-12} = \frac{1}{36\pi} 10^{-9} \text{ F/m}$$

$\therefore$ Coulomb's law is given as

$$F = \frac{Q_1 Q_2}{4\pi \epsilon_0 R^2}$$

3. If vector $\vec{r_1}$ locate Q_1 while $\vec{r_2}$ locate $\vec{Q_2}$, then $\vec{R}_{12} = \vec{r_2} - \vec{r_1}$ and

$$\hat{a}_{12} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|}.$$

4. The vector form of Coulomb's law is

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi \epsilon_0 R_{12}^2} \cdot \hat{a}_{12}$$

where, $\vec{F}_{12}$ = Force on Q_2 due to Q_1 (shown in Fig. 2.1.1)

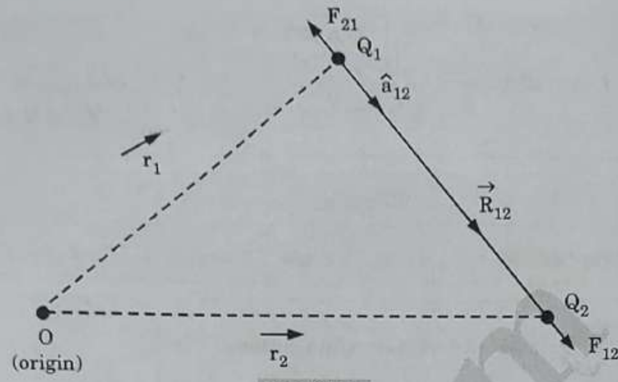


Fig. 2.1.1.

Que 2.2. **What do you mean by electric field intensity ?**

Answer

1. The electric field intensity $\vec{E}$ is the force per unit charge when placed in an electric field. Thus

$$\vec{E} = \frac{\vec{F}}{Q}$$

2. It is measured in Newton per Coulomb or volts per meter.
3. The electric field intensity at point $\vec{r}$ due to point charge located at $\vec{r}'$ is

$$\vec{E} = \frac{Q}{4\pi \epsilon_0 R^2}\hat{a}_R = \frac{Q(\vec{r}-\vec{r}')}{4\pi \epsilon_0 |\vec{r}-\vec{r}'|^3}$$

4. For N point charges $Q_1, Q_2 \ldots\ldots Q_N$ located at distance $\vec{r}_1, \vec{r}_2 \ldots\ldots\ldots \vec{r}_N$, the electric field intensity at point $\vec{r}$ is

$$\vec{E} = \frac{Q_1(\vec{r}-\vec{r}_1)}{4\pi \epsilon_0 |\vec{r}-\vec{r}_1|^3} + \frac{Q_2(\vec{r}-\vec{r}_2)}{4\pi \epsilon_0 |\vec{r}-\vec{r}_2|^3} + \ldots\ldots\ldots \frac{Q_N(\vec{r}-\vec{r}_N)}{4\pi \epsilon_0 |\vec{r}-\vec{r}_N|^3}$$

$$\vec{E} = \frac{1}{4\pi \epsilon_0}\sum_{k=1}^{N}\frac{Q_k(\vec{r}-\vec{r}_k)}{|\vec{r}-\vec{r}_k|^3}$$

Que 2.3. **Point charges 1 mC and – 2 mC are located at (3, 2, – 1) and (– 1, – 1, 4), respectively. Calculate the electric force on a 10 nC charge located at (0, 3, 1) and the electric field intensity at that point.**

AKTU 2021-22, Marks 10

Answer

1. $$F = \sum_{k=1,2}\frac{QQ_k}{4\pi\varepsilon_o R^2}a_R = \sum_{k=1,2}\frac{QQ_k(r-r_k)}{4\pi\varepsilon_o |r-r_k|^3}$$

$$= \frac{Q}{4\pi\varepsilon_o}\left\{\frac{10^{-3}[(0,3,1)-(3,2,-1)]}{|(0,3,1)-(3,2,-1)|^3} - \frac{2\times 10^{-3}[(0,3,1)-(-1,-1,4)]}{|(0,3,1)-(-1,-1,4)|^3}\right\}$$

$$= \frac{10^{-3}\times 10\times 10^{-9}}{4\pi\times\frac{10^{-9}}{36\pi}}\left[\frac{(-3,1,2)}{(9+1+4)^{3/2}} - \frac{2(1,4,-3)}{(1+16+9)^{3/2}}\right]$$

$$= 9\times 10^{-2}\left[\frac{(-3,1,2)}{14\sqrt{14}} + \frac{(-2,-8,6)}{26\sqrt{26}}\right]$$

$$F = -6.507a_x - 3.817a_y + 7.506a_z \text{ mN}$$

2. At that point, $$E = \frac{F}{Q}$$

$$= (-6.507, -3.817, 7.506)\times\frac{10^{-3}}{10\times 10^{-9}}$$

$$E = -650.7a_x - 381.7a_y + 750.6a_z \text{ kV/m}$$

Que 2.4. **Evaluate the electric field intensity in space due to charged finite length wire having uniform charge density.**

AKTU 2018-19, Marks 07

OR

Derive an expression for electric field intensity in space due to infinite length uniformly charged wire. **AKTU 2019-20, Marks 10**

Answer

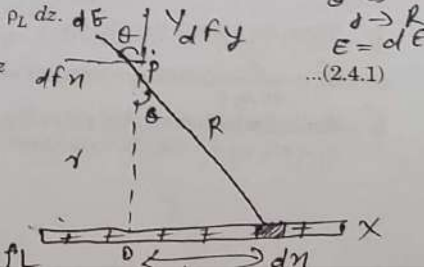
1. Let us consider a line charge with uniform charge density ρ_L extending from A to B along z-axis. The charge element dQ associated with element $dl = dz$ of the line is

θ → dq
d → R
E = dE

$$dQ = \rho_L\, dl = \rho_L\, dz.$$

2. The total charge, $$Q = \int_{z_A}^{z_B}\rho_L\, dz \qquad \ldots(2.4.1)$$

3. From Fig. 2.4.1,



dEy = dE cos θ
dEx = – dE sin θ
dEP = 2dE cos θ

$\tan\theta = \frac{x}{r}$

$x = r\tan\theta$

$\boxed{dx = r\sec^2\theta\, d\theta}$

$R^2 = r^2 + x^2$

$R^2 = r^2 + r^2\tan^2\theta$

$\boxed{R^2 = r^2\sec^2\theta}$

$\theta = \tan^{-1}\frac{x}{r}$

$x = 0 \quad \theta = 0$

$x = \infty \quad \theta = \frac{\pi}{2}$

And solve by the Int

$E = \frac{\rho_L}{2\pi\epsilon_0 r}$

And vector form

$$dl = dz'$$

$$\vec{R} = (x, y, z) - (0, 0, z')$$

$$= x\,\hat{a}_x + y\,\hat{a}_y + (z - z')\,\hat{a}_z$$

or

$$\vec{R} = \rho\hat{a}_\rho + (z - z')\,\hat{a}_z$$

$$R^2 = |\vec{R}|^2 = x^2 + y^2 + (z-z')^2 = \rho^2 + (z - z')^2$$

$$\frac{\hat{a}_R}{R^2} = \frac{\vec{R}}{|\vec{R}|^3} = \frac{\rho\hat{a}_\rho + (z - z')\hat{a}_z}{[\rho^2 + (z - z')^2]^{3/2}} \quad ...(2.4.2)$$

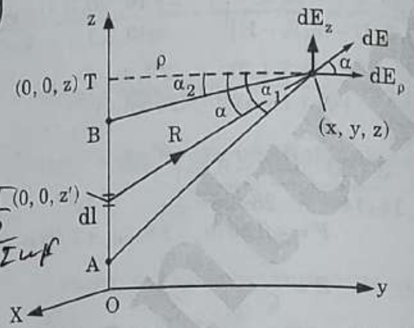


Fig. 2.4.1. Evaluation of the $\vec{E}$ field due to a line charge.

4. Putting the value of $\frac{\hat{a}_R}{R^2}$ and dl in $\vec{E} = \int_L \frac{\rho_L dl}{4\pi\epsilon_0 R^2}\hat{a}_R$

we get
$$\vec{E} = \frac{\rho_L}{4\pi\epsilon_0}\int \frac{\rho\hat{a}_\rho + (z - z')\hat{a}_z}{[\rho^2 + (z - z')^2]^{3/2}}\,dz' \quad ...(2.4.3)$$

5. As $\quad R = [\rho^2 + (z-z')^2]^{1/2} = \rho\sec\alpha$

and $\quad z' = OT - \rho\tan\alpha$

$\quad dz' = -\rho\sec^2\alpha\, d\alpha$

6. Putting values of R and dz' in eq. (2.4.3)

$$\vec{E} = \frac{-\rho_L}{4\pi\epsilon_0}\int_{\alpha_1}^{\alpha_2} \frac{\rho\sec^2\alpha[\cos\alpha\,\hat{a}_\rho + \sin\alpha\,\hat{a}_z]d\alpha}{\rho^2\sec^2\alpha}$$

$$\vec{E} = \frac{-\rho_L}{4\pi\epsilon_0\rho}\int_{\alpha_1}^{\alpha_2}[\cos\alpha\,\hat{a}_\rho + \sin\alpha\,\hat{a}_z]d\alpha \quad ...(2.4.4)$$

7. Thus, for a finite line charge,

$$\vec{E} = \frac{\rho_L}{4\pi\epsilon_0\rho}[-(\sin\alpha_2 - \sin\alpha_1)\hat{a}_\rho + (\cos\alpha_2 - \cos\alpha_1)\hat{a}_z] \quad ...(2.4.5)$$

8. For infinite line charge, point B is at $(0, 0, \infty)$ and A at $(0, 0, -\infty)$, so $\alpha_1 = \pi/2$, $\alpha_2 = -\pi/2$; the z component vanishes and eq. (2.4.5) becomes

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\hat{a}_\rho} \quad \text{vector form} \quad ...(2.4.6)$$

Que 2.5. **Find an expression for electric field intensity for an infinite sheet of charge.**

Answer

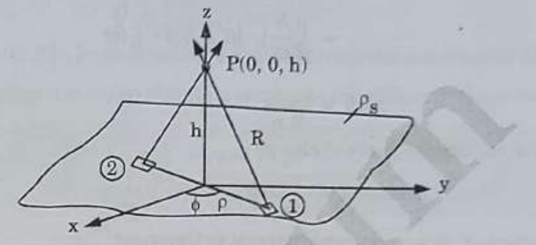


Fig. 2.5.1. Evaluation of the $\vec{E}$ field due to an infinite sheet of charge.

1. Consider an infinite sheet of charge in the x-y plane with uniform charge density ρ_S.
2. The charge associated with elemental area dS is
$$dQ = \rho_S\, dS \quad ...(2.5.1)$$
3. The $\vec{E}$ field at point P (0, 0, h) by the charge dQ on the elemental surface 1 is
$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 R^2}\hat{a}_R \quad ...(2.5.2)$$
4. From the Fig. 2.5.1,
$$\vec{R} = \rho(-\hat{a}_\rho) + h\,\hat{a}_z$$
$$R = |\vec{R}| = [\rho^2 + h^2]^{1/2}$$
$$\hat{a}_R = \frac{\vec{R}}{R},\; dQ = \rho_S\, dS = \rho_S\,\rho\, d\phi\, d\rho$$
5. Substituting the values of $\hat{a}_R$ and R in eq. (2.5.2)
$$d\vec{E} = \frac{\rho_S\,\rho\, d\phi\, d\rho[-\rho\hat{a}_\rho + h\,\hat{a}_z]}{4\pi\epsilon_0[\rho^2 + h^2]^{3/2}} \quad ...(2.5.3)$$
6. In the symmetry of the charge distributions, for every element 1, there is a corresponding element 2 whose contribution along $\hat{a}_\rho$ cancels that of element 1. Thus, the contribution to E_ρ add up to zero so that $\vec{E}$ has only z components.

7. This can be shown by replacing $\hat{a}_\rho$ with $\cos\phi\,\hat{a}_x + \sin\phi\,\hat{a}_y$. Integration $\cos\phi$ and $\sin\phi$ over $0 < \phi < 2\pi$ gives zero.

$$\therefore \quad \vec{E} = \int_S d\vec{E}_z = \frac{\rho_S}{4\pi\epsilon_0}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{\infty}\frac{\rho h\,d\rho\,d\phi}{[\rho^2+h^2]^{3/2}}\hat{a}_z$$

$$= \frac{\rho_S h}{4\pi\epsilon_0}2\pi\int_0^{\infty}[\rho^2+h^2]^{-3/2}\frac{1}{2}d(\rho^2)\hat{a}z$$

$$= \frac{\rho_s h}{2\epsilon_0}\{-[\rho^2+h^2]^{-1/2}\}_0^{\infty}\hat{a}z$$

$$\vec{E} = \frac{\rho_S\,\hat{a}_z}{2\epsilon_0}$$

8. For an infinite sheet of charge

$$\vec{E} = \frac{\rho_S}{2\epsilon_0}\hat{a}_n$$

where $\hat{a}_n$ is a unit vector normal to the sheet.

PART-2

Electric Flux Density.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.6. What is electric flux ? Explain the concept of electric flux density.

Answer

A. **Electric flux :** The total number of lines of force in any particular electric field is called electric flux. The unit of electric flux is coulombs (C) and represented by a symbol ψ.

B. **Electric flux density :**

1. The net flux (line of force) passing normal through the unit surface area is called the electric flux density.
2. It is a vector field quantity and represented by $\vec{D}$. It has a specific direction which is normal to the surface area.
3. Consider a sphere with a charge Q placed at its centre and there are no other charges present around. The total flux distributes radially around the charge is $\psi = Q$.

4. Let, ψ = Total flux
   S = Total surface area of sphere

   Then electric flux density is defined as

$$D = \frac{\psi}{S}\ \text{C/m}^2$$

5. It is also called displacement flux density or displacement density.
6. In vector form,

$$\vec{D} = \frac{d\psi}{dS}\hat{a}_n\ \text{C/m}^2$$

$d\psi$ = Total flux lines crossing normal through the differential area dS.

$\hat{a}_n$ = Unit vector in the direction normal to the differential surface area.

PART-3

Gausses Law-Maxwell's Equation.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.7. State Gauss's law. Deduce Maxwell's equation from Gauss's law.

OR

Prove that the net flux passing through any closed surface is equal to the charge enclosed by that surface.

Answer

1. Gauss's law states that the total electric flux ψ passing through any closed surface is equal to the total charge enclosed by that surface.
   Thus, $\psi = Q_{enc}$
2. Total flux passing through the entire closed surface is

$$\psi = \oint_S d\psi = \oint_S \vec{D}.d\vec{S} \qquad ...(2.7.1)$$

3. Such a closed surface over which the integration in the eq. (2.7.1) is carried out is called Gaussian surface.

$$\psi = \oint_S \vec{D}\cdot d\vec{S} = Q_{enc} = \text{Total charge enclosed.}$$

4. For volume charge distribution

$$\psi = Q_{enc} = \oint_S \vec{D}.d\vec{S} = \int_v \rho_v \cdot dv \qquad ...(2.7.2)$$

5. By applying divergence theorem in eq. (2.7.2)

$$\oint_S \vec{D}\cdot d\vec{S} = \oint_v \nabla\cdot\vec{D}\, dv \quad ...(2.7.3)$$

6. Comparing the volume integrals in eq. (2.7.2) and (2.7.3), we get

$$\rho_v = \nabla\cdot\vec{D}$$

7. This is the first Maxwell's equation.

Que 2.8. **Explain the applications of Gauss's law for point charge, line charge and coaxial cable.**

Answer

Applications of Gauss's Law :

i. **For point charge :**

1. Let a point charge Q is located at the origin. To determine $\vec{D}$ at a point P, a spherical surface containing point P is chosen. Thus, a spherical surface centered at the origin is the Gaussian surface in this case.
2. Since $\vec{D}$ is everywhere normal to the Gaussian surface, *i.e.*, $\vec{D} = D_r\hat{a}_r$. Applying Gauss's law gives

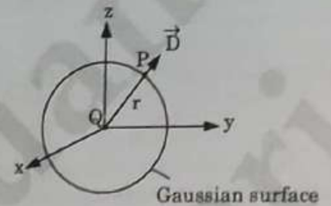


Fig. 2.8.1. Gaussian surface about a point charge.

$$Q = \oint_S \vec{D}\cdot d\vec{S} = D_r\oint_S dS = D_r 4\pi r^2$$

where, $\oint_S dS = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} r^2\sin\theta\, d\theta\, d\phi = 4\pi r^2$

is the surface area of the Gaussian surface.

$$\therefore \quad \boxed{\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r}$$

ii. **For infinite line charge :**

1. Let the infinite line of uniform charge ρ_L C/m lies along the z-axis. To determine $\vec{D}$ at point P, a cylindrical surface containing point P is chosen.
2. As, the electric flux density $\vec{D}$ is constant and normal to the cylindrical surface,

$$\therefore \quad \vec{D} = D_\rho\hat{a}_\rho$$

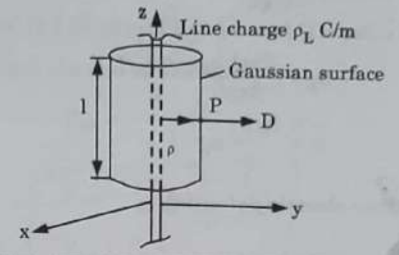


Fig. 2.8.2. Gaussian surface about an infinite line charge.

3. Applying Gauss's law to the length l of the line

$$\rho_L l = Q = \oint_S \vec{D}.d\vec{S} = D_\rho\oint_S dS = D_\rho\, 2\pi\rho l$$

where $\oint dS = 2\pi\rho l$ is surface area of Gaussian surface.

Thus, $$\vec{D} = \frac{\rho_L}{2\pi\rho}\hat{a}_\rho$$

iii. **For coaxial cable :**

1. Let us consider two coaxial cylindrical conductors, having 'a' as the inner radius and 'b' as the outer radius.
2. Let us assume a charge distribution of ρ_S on the outer surface of the inner conductor. Here, a circular cylinder of length L and radius ρ, where $a < \rho < b$ is chosen as the Gaussian surface.

$$\therefore \quad Q = \vec{D}_S 2\pi\rho L \quad ...(2.8.1)$$

3. Thus, the total charge on a length L of the inner conductor is

$$Q = \int_{z=0}^{L}\int_{\phi=0}^{2\pi} \rho_S\, a\, d\phi\, dz = 2\pi a L\rho_S \quad ...(2.8.2)$$

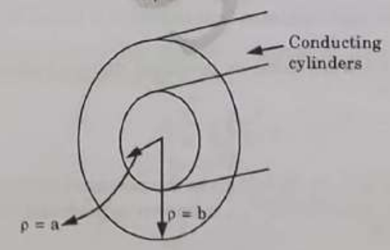


Fig. 2.8.3.

3. It also implies that no net work is done in moving a charge along a closed path in an electrostatic field.
4. Applying Stoke's theorem to eq. (2.10.1)

$$\oint_L \vec{E}.d\vec{l} = \int_S (\nabla \times \vec{E}).d\vec{S} = 0$$

or $$\nabla \times \vec{E} = 0 \quad ...(2.10.2)$$

5. Eq. (2.10.1) and (2.10.2) referred to as Maxwell's equation for static electric fields.

As $$V = -\int \vec{E}.d\vec{l}$$

or $$dV = -\vec{E}.d\vec{l}$$

$$dV = -E_x\,dx - E_y\,dy - E_z\,dz \quad ...(2.10.3)$$

6. Since a total charge in $V(x, y, z)$ is the sum of partial charges with respect to x, y, z variables :

$\therefore$ $$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz \quad ...(2.10.4)$$

7. Comparing eq. (2.10.3) and (2.10.4), we get

$$E_x = -\frac{-\partial V}{\partial x}, E_y = \frac{-\partial V}{\partial y}, E_z = \frac{-\partial V}{\partial z}$$

$\therefore$ $$\vec{E} = -\nabla V \quad ...(2.10.5)$$

8. The eq. (2.10.5) indicates that the electric field intensity is the gradient of V and negative sign shows that the direction of $\vec{E}$ is opposite to the direction in which V increases.

Que 2.11. **Two point charges – 4 nC and 5 nC are placed at (2, – 1, 3) and (0, 4, – 2) respectively, find the potential at (1, 0, 1), assuming zero potential at infinity. Right angle triangle. Find electric forces at the corners of the triangle.**

AKTU 2018-19, Marks 07

Answer

A. **Calculation of electric potential :**

1. Let $$Q_1 = -4\text{ nC}, Q_2 = 5\text{ nC}$$

$$V(r) = \frac{Q_1}{4\pi\varepsilon_0 |r - r_1|} + \frac{Q_1}{4\pi\varepsilon_0 |r - r_2|} + C_0$$

2. If $$V(\infty) = 0, C_0 = 0$$

$$|r - r_1| = |(1, 0, 1) - (2 - 1, 3)| = |(-1, 1, -2)| = \sqrt{6}$$

$$|r - r_2| = |(1, 0, 1) - (0, 4, -2)| = |(1, -4, 3)| = \sqrt{26}$$

3. Hence $$V(1, 0, 1) = \frac{10^{-9}}{4\pi \times \frac{10^{-9}}{36\pi}}\left[\frac{-4}{\sqrt{6}} + \frac{5}{\sqrt{26}}\right] = -5.872\text{ V}$$

B. **Calculation of electric force :**

1. Assume a charge of 1 nC at point $C(x, y, z)$.

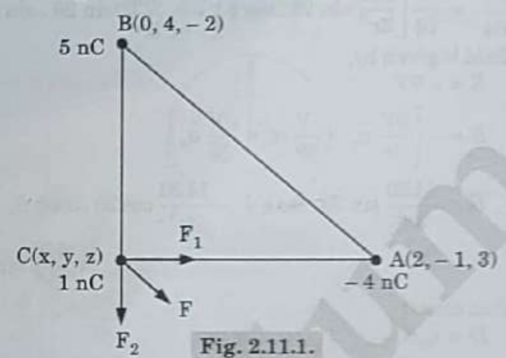


Fig. 2.11.1.

2. Force on C due to A,

$$F_1 = \frac{(1 \times 10^{-9})(4 \times 10^{-9})}{4\pi\varepsilon_0[(x-2)^2 + (y+1)^2 + (z-3)^2]}$$

This force acts along CA.

3. Force on C due to B,

$$F_2 = \frac{(1 \times 10^{-9})(5 \times 10^{-9})}{4\pi\varepsilon_0[(x-0)^2 + (y-4)^2 + (z+2)^2]}$$

This force acts along BC.

4. Resultant force acts at point C,

$$F = \sqrt{F_1^2 + F_2^2}$$

Que 2.12. **Given the potential $V = \frac{560}{3r^2}\sin 2\theta \cos\phi$. Find the electric flux density D at (2, 90°, 0). Also calculate the work done in moving a 10 μC charge from point A(1, 30°, 120°) to B(2, 60°, 30°).**

AKTU 2021-22, Marks 10

Answer

1. Potential $$V = \frac{560}{3r^2}\sin 2\theta \cos\phi$$
2. The electric field is given by
$$E = -\nabla V$$

3. $$\frac{\partial V}{\partial r} = \frac{\partial}{\partial r}\left[\frac{560}{3r^2}\sin 2\theta \cos\phi\right] = -\frac{560 \times 2}{3r^3}\sin 2\theta\cos\phi$$

4. $$\frac{\partial V}{\partial \theta} = \frac{\partial}{\partial \theta}\left[\frac{560}{3r^2}\sin 2\theta \cos\phi\right] = \frac{560 \times 2}{3r^2}\cos 2\theta \,.\, \cos\phi$$

5. $$\frac{\partial V}{\partial \phi} = \frac{\partial}{\partial \phi}\left[\frac{560}{3r^2}\sin 2\theta \,.\, \cos\phi\right] = -\frac{560}{3r^2}\sin 2\theta \,.\, \sin\phi$$

6. Electric field is given by,
$$E = -\nabla V$$
$$E = -\left[\frac{\partial V}{\partial r}\hat{a}_r + \frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{\partial V}{\partial \phi}\hat{a}_\phi\right]$$

7. $$E = \frac{1120}{3r^3}\sin 2\theta \,.\, \cos\phi\,\hat{a}_r - \frac{1120}{3r^2}\cos 2\theta \,.\, \cos\phi\,\hat{a}_\theta + \frac{560}{3r^2}\sin 2\theta \,.\, \sin\phi\,\hat{a}_\phi$$

8. Electric flux density,
$\because$ $$D = \varepsilon_0 E$$
$$= \varepsilon_0\left[\frac{1120}{3r^3}\sin 2\theta \,.\, \cos\phi\,\hat{a}_r - \frac{1120}{3r^2}\,.\cos 2\theta\cos\phi\,\hat{a}_\theta + \frac{560}{3r^2}\sin 2\theta \sin\phi\,\hat{a}_\phi\right]$$

At ($r = 2, \theta = 90°, \phi = 0$)
$$= \varepsilon_0\left[\frac{1120}{3(2)^3}\sin 180° \,.\, \cos 0°\,\hat{a}_r - \frac{1120}{3(2)^2}\,.\cos 180° \,.\, \cos 0°\,\hat{a}_\theta + \frac{560}{3(2)^2}\sin 180° \,.\, \sin 0°\hat{a}_\phi\right]$$
$$= \varepsilon_0\left[0 + 93.33\,\hat{a}_\theta + 0\right] = 93.33\,\varepsilon_0\,\hat{a}_\theta \text{ C/m}^2$$

9. Work done,
Since V is known, so
$$W = -Q\int_A^B E \,.\, dl = QV_{AB}$$
$$= Q(V_B - V_A)$$
$$= 10 \times 10^{-6}\left(\frac{560}{3 \times 4}\sin 120 \,.\, \cos 30 - \frac{560}{3 \times 1}\sin 60.\cos 120\right)$$
$$= 10 \times 10^{-6}\,(34.51 + 80.26)$$
$$= 1147.76 \times 10^{-6}\text{ J} = 1147.76\ \mu\text{J}.$$

Que 2.13. **Find the potential function and electric field intensity for the region between two concentric right circular cylinder where $V = V_o$ at $r = a$ and $V = 0$ at $r = b$ ($b > a$).** **AKTU 2017-18, Marks 07**

Answer

1. Consider the spherical shells (top of cylinders) and assume that V depends only on 'r'.
2. Here Laplace equation becomes
$$\nabla^2 = \frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{dV}{dr}\right] = 0$$

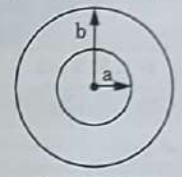


Fig. 2.13.1.

3. Since $r \neq 0$ is the region of interest,
$$\frac{d}{dr}\left[r^2\frac{dV}{dr}\right] = 0$$
Integrating once gives,
$$r^2\frac{dV}{dr} = A$$
or $$\frac{dV}{dr} = \frac{A}{r^2}$$
Integrating again gives,
$$V = -\frac{A}{r} + B \quad ...(2.13.1)$$
A and B are integration constants,
4. When $r = b$, $V = 0 = -\dfrac{A}{b} + B$
$$B = \frac{A}{b} \quad ...(2.13.2)$$
5. Substituting eq. (2.13.2) in eq. (2.13.1), we get
$$V = A\left[\frac{1}{b} - \frac{1}{r}\right] \quad ...(2.13.3)$$
6. Also, when $r = a$, $V = V_0$
$$V_0 = A\left[\frac{1}{b} - \frac{1}{a}\right]$$
$$A = -\frac{V_0}{\left[\dfrac{1}{a} - \dfrac{1}{b}\right]}$$
7. Substituting value of A in eq. (2.13.3), we get

$$V = \frac{V_0\left[\frac{1}{r}-\frac{1}{b}\right]}{\left[\frac{1}{a}-\frac{1}{b}\right]}$$

$$E = -\Delta V = -\frac{dV}{dr}\hat{a}_r = -\frac{A}{r^2}\hat{a}_r = \frac{V_0}{r^2\left[\frac{1}{a}-\frac{1}{b}\right]}.$$

PART-4

Electric Dipole and Flux Line.

Questions-Answers
Long Answer Type and Medium Answer Type Questions

Que 2.14. **Write short notes on following :**

i. Electric dipole
ii. Flux lines

Answer

i. Electric dipole :

1. An electric dipole is formed when two point charges of equal magnitude but opposite sign are separated by a small distance. Let us consider a dipole as shown in the Fig. 2.14.1.

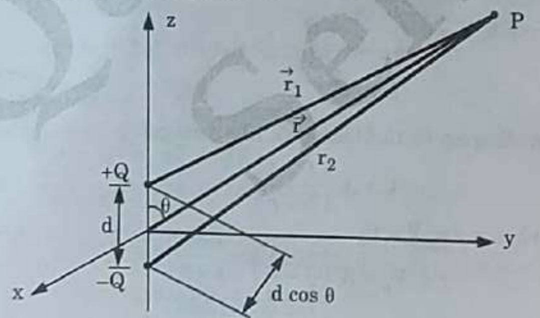


Fig. 2.14.1.

2. The potential at point $P(r, \theta, \phi)$ is given by

$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{\vec{r}_1}-\frac{1}{\vec{r}_2}\right] = \frac{Q}{4\pi\epsilon_0}\left[\frac{\vec{r}_2-\vec{r}_1}{\vec{r}_1\vec{r}_2}\right] \quad ...(2.14.1)$$

where $\vec{r}_1$ and $\vec{r}_2$ are the distances between P and $+Q$, and P and $-Q$.

3. If $r >> d$, $\vec{r}_2 - \vec{r}_1 = d\cos\theta$, $\vec{r}_1\,\vec{r}_1 = r^2$ then eq. (2.14.1) becomes

$$V = \frac{Q}{4\pi\epsilon_0}\frac{d\cos\theta}{r^2} \quad ...(2.14.2)$$

4. Since $d\cos\theta = \vec{d}.\hat{a}_r$ where $\vec{d} = d\hat{a}_z$ and if we consider $\vec{p} = Q\vec{d}$ as dipole moment then eq. (2.14.2) may be written as

$$V = \frac{\vec{p}.\hat{a}_r}{4\pi\epsilon_0 r^2} \quad ...(2.14.3)$$

5. If the dipole center is not at the origin but at r', eq. (2.14.3) becomes

$$V(r) = \frac{\vec{p}(\vec{r}-\vec{r}')}{4\pi\epsilon_0|\vec{r}-\vec{r}'|^3} \quad ...(2.14.4)$$

6. The electric field due to the dipole with center at the origin is given as

$$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial\theta}\hat{a}_\theta\right]$$

$$\vec{E} = \frac{Qd\cos\theta}{2\pi\epsilon_0 r^3}\hat{a}_r + \frac{Qd\sin\theta}{4\pi\epsilon_0 r^3}\hat{a}_\theta$$

$$\therefore \quad \vec{E} = \frac{\vec{p}}{4\pi\epsilon_0 r^3}(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta) \quad [\text{Here } \vec{p} = Q\vec{d}\,]$$

ii. Electric flux line : An electric flux line is an imaginary path or a line drawn in such a way that its direction at any point is the direction of the electric field at that point. These are the lines to which electric flux density $\vec{D}$ is tangential at every point.

PART-5

Energy Density in Electrostatic Fields.

Questions-Answers
Long Answer Type and Medium Answer Type Questions

Que 2.15. **Derive energy density in electrostatic field.**

Answer

1. Consider the volume charge distribution with uniform charge density ρ_v C/m^3. Therefore total energy stored is

$$W_E = \frac{1}{2}\int_v \rho_v V\, dv \quad ...(2.15.1)$$

2. Since, $\rho_v = \nabla \cdot \vec{D}$

$$\therefore \quad W_E = \frac{1}{2}\int_v (\nabla \cdot \vec{D})\, V\, dv \quad ...(2.15.2)$$

3. For any vector $\vec{A}$ and scalar V, the identity

$$\nabla \cdot V\vec{A} = \vec{A}\cdot\nabla V + V(\nabla\cdot\vec{A})$$

or
$$(\nabla\cdot\vec{A})V = \nabla\cdot V\vec{A} - \vec{A}\cdot\nabla V \quad ...(2.15.3)$$

Applying the identity in eq. (2.15.3) to eq. (2.15.2), we get

$$W_E = \frac{1}{2}\int_v (\nabla\cdot V\vec{D})\, dv - \frac{1}{2}\int_v (\vec{D}\cdot\nabla V)\, dv \quad ...(2.15.4)$$

4. By applying divergence theorem to the first term on the right-hand side of eq. (2.15.4), we have

$$W_E = \frac{1}{2}\oint_S (V\vec{D})\cdot dS - \frac{1}{2}\int_v (\vec{D}\cdot\nabla V)\, dv \quad ...(2.15.5)$$

5. We know that V varies as $1/r$ and $\vec{D}$ as $1/r^2$ for point charges; V varies as $1/r^2$ and $\vec{D}$ as $1/r^3$ for dipoles; and so on. Hence, $V\vec{D}$ in the first term on the right-hand side of eq. (2.15.5) must vary at least as $1/r^3$ while dS varies as r^2.

6. Consequently, the first integral in eq. (2.15.5) must tend to zero as the surface S becomes large. Hence, eq. (2.15.5) reduces to

$$W_E = -\frac{1}{2}\int_v (\vec{D}\cdot\nabla V)\, dv = \frac{1}{2}\int_v (\vec{D}\cdot\vec{E})\, dv \quad ...(2.15.6)$$

7. Since, $\vec{E} = -\nabla V$ and $\vec{D} = \epsilon_0 \vec{E}$

$$W_E = \frac{1}{2}\int_v \vec{D}\cdot\vec{E}\, dv = \frac{1}{2}\int_v \epsilon_0 E^2\, dv \quad ...(2.15.7)$$

8. From this, we can define electrostatic energy density w_E (in J/m^3) as

$$w_E = \frac{dW_E}{dv} = \frac{1}{2}\vec{D}\cdot\vec{E} = \frac{1}{2}\epsilon_0 E^2 = \frac{D^2}{2\epsilon_0} \quad ...(2.15.8)$$

So eq. (2.15.6) may be written as

$$W_E = \int_v w_E\, dv \quad ...(2.15.9)$$

Que 2.16. **Derive the mathematical expression for energy stored in electric field. If $V = yx^2 + zx + xy$ V. Do the analysis of $\vec{E}$ at (2, 3, 7) and the electrostatic energy stored in a cube of side 4 m centered at origin.**

AKTU 2021-22, Marks 10

Answer

A. **Energy stored in electric field :** Refer Q. 2.15, Page 2–18B, Unit-2.

B. **Numerical :**

1. $V = yx^2 + zx + xy$ V

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}[yx^2 + zx + xy] = 2yx + z + y$$

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}[yx^2 + zx + xy] = x^2 + x$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}[yx^2 + zx + xy] = x$$

2. The electric field is given by

$$E = -\nabla V = -\left[\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right]$$

$$= -[(2yx + z + y)\hat{a}_x + (x^2 + x)\hat{a}y + x\hat{a}_z]$$

$$= -(2yx + z + y)\hat{a}_x - (x^2 + x)\hat{a}y - x\hat{a}_z \quad ...(2.16.1)$$

$$E|_{(2,3,7)} = -(2\times 3\times 2 + 7 + 3)\hat{a}_x - (4+2)\hat{a}y - 2\hat{a}_z$$

$$= 2\hat{a}_x - 6\hat{a}_y - 2\hat{a}_z \text{ V/m}$$

3. Taking mod of $|E|$ in eq. (2.16.1)

$$|E| = \sqrt{(2yx + z + y)^2 + (x^2 + x)^2 + x^2}$$

$$|E|^2 = x^4 + 2x^3 + 2x^2 + 4y^2x^2 + y^2 + z^2 + 2yz + 4y^2x + 4xyz$$

The cube of 4 m side is centered at origin, so $-2 \le x, y, z \le 2$.

4. The energy stored is given by,

$$W_E = \frac{1}{2}\varepsilon_0 \int_V |E|^2\, dV$$

$$= \frac{1}{2}\varepsilon_0 \int_{-2}^{2}\int_{-2}^{2}\int_{-2}^{2} (x^4 + 2x^3 + 2x^2 + 4y^2x^2 + y^2 + z^2 + 2yz + 4y^2x + 4xyz)\, dxdydz$$

$$= \frac{1}{2}\varepsilon_0\, [1010.13] = 505.6\varepsilon_0 \text{ J}$$

Que 2.17. **Point charges $Q_1 = 1$ nC, $Q_2 = -2$ nC, $Q_3 = 3$ nC and $Q_4 = -4$ nC are positioned one at a time in that order at (0, 0, 0), (1, 0, 0), (0, 0, –1) and (0, 0, 1) respectively. Calculate the energy in the system after each charge is positioned.**

AKTU 2017-18, Marks 07

Answer

1. When Q_1 is placed, the work done is zero as $\vec{E} = 0$, hence $W_1 = 0$ J.
2. When Q_2 is placed, there is field of Q_1 present.

$$\therefore \quad W_2 = Q_2 V_{21} = Q_2 \times \frac{Q_1}{4\pi\varepsilon_0 R_{21}}$$

$$R_{21} = 1,\ W_2 = \frac{-2\times10^{-9}\times1\times10^{-9}}{4\pi\times8.854\times10^{-12}} = -17.9754 \text{ nJ}$$

3. When Q_3 is placed, there is field due to Q_1 and Q_2 both.

$$\therefore \quad W_3 = Q_3 V_{31} + Q_3 V_{32}$$

$$\therefore \quad W_3 = \left[\frac{Q_1}{4\pi\varepsilon_0 R_{31}} + \frac{Q_2}{4\pi\varepsilon_0 R_{32}}\right] \text{ and } R_{31} = 1, R_{32} = \sqrt{2}$$

$$= \frac{3\times10^{-9}}{4\pi\varepsilon_0}\left[\frac{1\times10^{-9}}{1} - \frac{2\times10^{-9}}{\sqrt{2}}\right] = -11.168 \text{ nJ}$$

4. When Q_4 is placed, there is field due to Q_1, Q_2 and Q_3

$$W_4 = Q_4V_{41} + Q_4V_{42} + Q_4V_{43}$$

$$= Q_4\left[\frac{Q_1}{4\pi\varepsilon_0 R_{41}} + \frac{Q_2}{4\pi\varepsilon_0 R_{42}} + \frac{Q_3}{4\pi\varepsilon_0 R_{43}}\right]$$

and $\quad R_{41} = 1, R_{42} = \sqrt{2}, R_{43} = 2$

$$= \frac{-4\times10^{-9}}{4\pi\varepsilon_0}\left[\frac{1\times10^{-9}}{\sqrt{2}} + \frac{3\times10^{-9}}{2}\right] = -39.03 \text{ nJ}$$

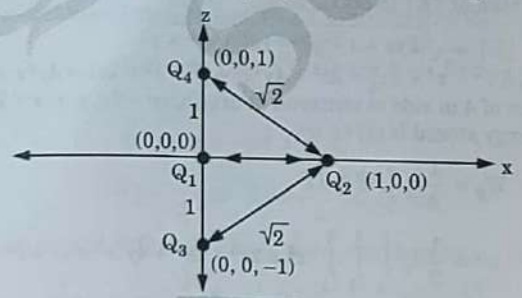


Fig. 2.17.1.

5. Total energy, $\quad W_E = W_1 + W_2 + W_3 + W_4$
$= 0 - 17.9754 - 11.168 - 39.035 = -68.178$ nJ

PART-6

Electric Field in Material Space : Properties of Materials, Convection and Conduction Currents.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.18. **Discuss some properties of material.**

Answer

1. Materials may be classified in terms of their conductivity σ, in mho per meter (℧ /m) or Siemens per meter (S/m).
2. The conductivity of a material usually depends on temperature.
3. A material with high conductivity ($\sigma >> 1$) is termed as metal whereas one with low conductivity ($\sigma << 1$) is termed as insulator.
4. At temperatures near absolute zero ($T = 0$ K), some conductor exhibit infinite conductivity and are called as superconductors.

Que 2.19. **Write short notes on :**

i. Convection current
ii. Conduction current

Answer

i. Convection current :

1. Convection current does not involve conductors and consequently does not satisfy Ohm's law. It occurs when current flows through an insulating medium such as liquid, rarefied gas, or a vacuum.
2. Let us consider a filament, if there is a flow of charge of density ρ_v, at velocity $\vec{u} = u_y\,\hat{a}_y$ then the current through the filament is

$$\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \Delta S \frac{\Delta y}{\Delta t} = \rho_v \Delta S\, u_y$$

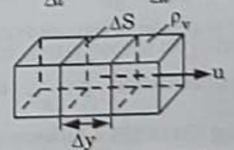


Fig. 2.19.1. Current in a filament.

3. The y-directed current density J_y is given by

$$J_y = \frac{\Delta I}{\Delta S} = \rho_v u_y$$

Hence, in general

$$\therefore \quad \vec{J} = \rho_v \vec{u}$$

Here, I = Convection current and J = Convection current density.

ii. Conduction current :

1. Conduction current requires a conductor. A conductor consists of large number of free electrons that provide conduction current due to an impressed electric field.
2. When an electric field $\vec{E}$ is applied, the force on an electron with charge '$-e$' is given as

$$\vec{F} = -e\vec{E} = \frac{m\vec{u}}{\tau}$$

3. If an electron with mass m is moving in an electric field $\vec{E}$ with an average drift velocity $\vec{u}$, according to Newton's law, the average change in momentum of the free electron must match the applied force. Thus,

$$\therefore \quad \frac{m\vec{u}}{\tau} = -e\vec{E}$$

$$\vec{u} = \frac{-e\tau\vec{E}}{m}$$

where m = Mass of electron, $\vec{u}$ = Average drift velocity and τ is the average time interval between collisions.

4. If there are n electrons per unit volume, the electron charge density,

$$\rho_v = -ne$$

5. Conduction current density is

$$\vec{J} = \rho_v \vec{u} = \frac{ne^2\tau}{m}\vec{E} = \sigma\vec{E}$$

where, $\sigma = ne^2\tau/m$ is the conductivity of conductor.

Que 2.20. **If the current density, $\vec{J} = \frac{1}{r^2}(\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta)$ A/m². Find the current passing through a sphere radius of 1.0 m.**

Answer

1. Given, $\vec{J} = \frac{1}{r^2}(\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta)$

$$r = 1\text{ m}, 0 \le \theta \le \pi$$

2. We know, $I = \int \vec{J}\cdot d\vec{S}$

where, $d\vec{S} = r^2 \sin\theta\, d\phi\, d\theta\, \hat{a}_r$

$$\therefore \quad I = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{1}{r^2}(\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta).r^2 \sin\theta\, d\phi\, d\theta\,\hat{a}_r\Big|_{r=1}$$

$$= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{1}{r^2}\cos\theta\, r^2 \sin\theta\, d\phi\, d\theta\Big|_{r=1}$$

$$= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{2.\cos\theta\sin\theta\, d\phi\, d\theta}{2}\Big|_{r=1}$$

$$= 2\pi\int_{\theta=0}^{\pi} \frac{\sin 2\theta}{2}\, d\theta = \pi\int_{\theta=0}^{\pi} \sin 2\theta = \pi\left[\frac{-\cos 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{\pi}{2}[-\cos(2\pi) + \cos(0)] = \frac{\pi}{2}[-1+1] = 0$$

PART-7

Polarization in Dielectrics, Dielectric-Constants.

Questions-Answers
Long Answer Type and Medium Answer Type Questions

Que 2.21. **Explain the phenomena of polarization and its types.**

AKTU 2017-18, Marks 07

OR

Discuss polarization in dielectric medium.

Answer

A. Polarization : It is defined as the property of electromagnetic radiations where the relationship between the direction and magnitude of the vibrating electric field is explained.

B. Types of polarization :

1. **Dielectric polarization :** This type of polarization occurs when a dipole moment is formed in an insulating material as an external electric field is applied there is a change in the behaviour which is known as dielectric polarization.
2. **Ionic polarization :** When elements like NaCl and KCl contribute to the relative permittivity, ionic polarization occurs. In this polarization, the net electric field is zero.
3. **Orientational polarization :** This occurs due to the permanent dipole moment in a material. It occurs in elements such as HCl and H2O.

C. Polarization in dielectric medium :

1. The charges in dielectrics are bound by the finite forces and hence called bound charges.

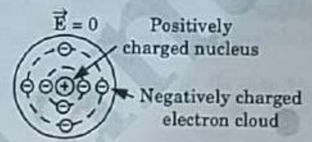


Fig. 2.21.1. Unpolarized atom of a dielectric.

2. The process due to which separation of bound charges results to produce electric dipoles, under the influence of electric field, is called polarization of dielectric.
3. When electric field $\vec{E}$ is applied, the symmetrical distribution of charges gets disturbed. The charges experience a force $\vec{F} = Q\vec{E}$.
4. So, there is separation between the nucleus and the centre of the electron cloud. Such an atom is called polarized atom.

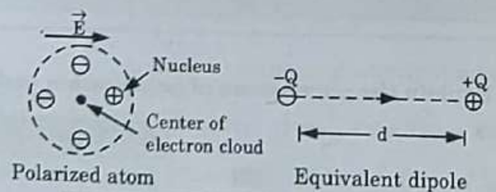


Fig. 2.21.2. Polarization of a molecule.

Mathematical expression for polarization :

1. Electric dipole moment, $\vec{p} = Q\vec{d}$ ...(2.21.1)

   where, Q = Magnitude of one of the two charges

   $\vec{d}$ = Distance vector from negative to positive charge

2. Let, n = Number of dipoles per unit volume

   Δv = Total volume of dielectric

   $N = n.\Delta v$ = Total dipoles.

   Total dipole moment, $\vec{P}_{total} = \sum_{i=1}^{N} Q_i \vec{d}_i$

3. The polarization $\vec{P}$ is defined as the total dipole moment per unit volume.

$$\vec{P} = \lim_{\Delta v \to 0} \frac{\sum_{i=1}^{n} Q_i \vec{d}_i}{\Delta v}$$

It is measured in Coulombs per square meter (C/m^2).

Que 2.22. **What do you mean by dielectric constant ?**

Answer

1. It is also known as relative permittivity, denoted by ϵ_r. It is the ratio of the permittivity of dielectric to that of free space.
2. It is given by

$$\epsilon_r = 1 + \chi_e = \frac{\epsilon}{\epsilon_0}$$

where, ϵ = Permittivity of dielectric

ϵ_0 = Permittivity of free space.

Here ϵ_r and χ_e are dimensionless whereas ϵ and ϵ_0 are in Farads per meter.

3. For free space and non-dielectric materials, $\epsilon_r = 1$.

PART-8

Continuity Equation and Relaxation Time.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.23. **Derive and explain continuity equation for electrostatic in detail.**

AKTU 2018-19, Marks 07

Answer

+ f +
According to charge conservation law
Rate of decrease of charge = Net outward current

1. This equation of current is based on the principle of conservation of charge. The principle of conservation of charge states that "The charges can neither be created nor be destroyed." $I_{out} = \oint_s \vec{J}\,\vec{ds}$

$$I = \oint_S \vec{J}.d\vec{S} \quad \text{...(2.23.1)}$$

2. Now, let Q_i = Charge within the closed surface.

$\therefore$ $-\dfrac{dQ_i}{dt}$ = Rate of decrease of charge inside the closed surface.

3. According to principle of conservation of charge, this rate of decrease is same as rate of outward flow of charge. Flow of charge is current.

So, $$\boxed{I = \oint_S \vec{J}.d\vec{S} = -\frac{dQ_i}{dt}} \quad \text{...(2.23.2)}$$

4. Eq. (2.23.2) is the integral form of the continuity equation of current. Now, using the divergence theorem,

$$\oint_S \vec{J}.d\vec{S} = \int_v (\nabla\cdot\vec{J})\, dv \quad \text{...(2.23.3)}$$

So, $$-\frac{dQ_i}{dt} = \int_v (\nabla\cdot\vec{J})\, dv$$

But $$\frac{-d}{dt}Q_i = -\frac{d}{dt}\int_v \rho_v dv = -\int \frac{\partial\rho_v}{\partial t}\, dv \quad \text{...(2.23.4)}$$

where, ρ_v = Volume charge density

5. Substituting eq. (2.23.3) and (2.23.4) in eq. (2.23.2),

$$\int_v \nabla.\vec{J}\, dv = -\int_v \frac{\partial\rho_v}{\partial t}\, dv$$

$$\Rightarrow \quad \boxed{\nabla\cdot\vec{J} = -\frac{\partial\rho_v}{\partial t}}$$

6. This is the point form or differential form of the continuity equation of the current.

Que 2.24. **Write a short note on relaxation time.**

Answer

1. Let us consider a conducting material which is linear and homogeneous. The current density for this type of material is,

$$\vec{J} = \sigma\vec{E} \quad \text{(ohm law)}$$

But $$\vec{D} = \epsilon\vec{E} \Rightarrow \vec{E} = \frac{\vec{D}}{\epsilon}$$

$$\therefore \quad \vec{J} = \sigma\frac{\vec{D}}{\epsilon} = \frac{\sigma}{\epsilon}\vec{D} \quad \text{(Gauss law)}$$

2. The differential form of the continuity equation states that,

$$\nabla\cdot\vec{J} = -\frac{\partial\rho_v}{\partial t}$$

$$\nabla\cdot\left(\frac{\sigma}{\epsilon}\vec{D}\right) = -\frac{\partial\rho_v}{\partial t}$$

$$\frac{\sigma}{\epsilon}\nabla\cdot\vec{D} = -\frac{\partial\rho_v}{\partial t}$$

But $$\nabla\cdot\vec{D} = \rho_v$$

$$\therefore \quad \frac{\sigma\rho_v}{\epsilon} = -\frac{\partial\rho_v}{\partial t}$$

$$\therefore \quad \frac{\partial\rho_v}{\partial t} + \frac{\sigma}{\epsilon}\rho_v = 0 \quad \text{...(2.24.1)}$$

3. Eq. (2.24.1) is the differential equation in ρ_v whose solution is given by,

$$\rho_v = \rho_0\, e^{-(\sigma/\epsilon)t} = \rho_0\, e^{-t/\tau} \qquad \rho_v = \rho_0\, e^{-t/\tau}$$

where, ρ_0 = Charge density at $t = 0$

4. This shows that the charge density decays exponentially with a time constant $\tau = \epsilon/\sigma$ sec. This time is called relaxation time.

5. The relaxation time (τ) is defined as the time required by the charge density to decay to 36.8 % of its initial value.

τ = Relaxation time = ϵ/σ sec.

PART-9

Boundary Conditions.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.25. **Explain the term boundary conditions.**

Answer

1. If the field exists in a region consisting of two different media, the conditions that the field must satisfy at the interface separating either media are called boundary conditions.
2. To determine the boundary conditions, Maxwell's equations are used. *i.e.,*

$$\oint \vec{E}.d\vec{l} = 0 \text{ and} \oint \vec{D}.d\vec{S} = Q_{enc}$$

3. The electric field intensity $\vec{E}$ can be decomposed into two orthogonal components ; *i.e.,* $E = E_t + E_n$

   where E_t and E_n are the tangential and normal components of $\vec{E}$ to the interface.
4. Some boundary conditions at an interface are :
   i. Dielectric (ϵ_{r1}) and dielectric (ϵ_{r2})
   ii. Conductor and dielectric
   iii. Conductor and free space.

Que 2.26. **Explain dielectric-dielectric boundary condition.**

Answer

1. Consider the $\vec{E}$ field existing in a region that consists of two different dielectrics characterized by $\epsilon_1 = \epsilon_0\epsilon_{r1}$ and $\epsilon_2 = \epsilon_0\epsilon_{r2}$ as shown in Fig. 2.26.1.

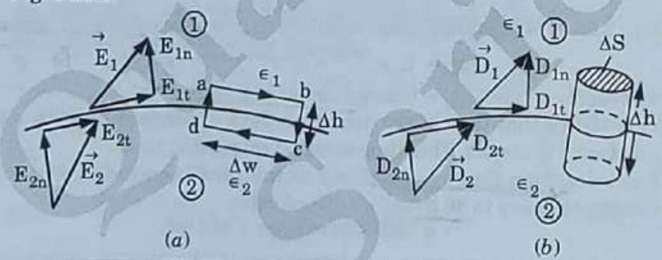


Fig. 2.26.1. Dielectric-dielectric boundary : (*a*) Determining $E_{1t} = E_{2t}$, (*b*) Determining $D_{1n} = D_{2n}$.

2. The fields $\vec{E}_1$ and $\vec{E}_2$ in media 1 and 2, can be decomposed as

$$\vec{E}_1 = \vec{E}_{1t} + \vec{E}_{1n} \quad ...(2.26.1)$$

$$\vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n} \quad ...(2.26.2)$$

3. Applying $\oint \vec{E}.d\vec{l} = 0$ to the closed path *abcda* and assuming that the path is very small with respect to the spatial variation of $\vec{E}$. We obtain

$$0 = E_{1t}\Delta w - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\Delta w + E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2} \quad ...(2.26.3)$$

where $E_t = |\vec{E}_t|$ and $E_n = |\vec{E}_n|$ and term $\frac{\Delta h}{2}$ cancels.

$\therefore$ Eq. (2.26.3) becomes, $0 = (E_{1t} - E_{2t})\Delta w$ ...(2.26.4)

As $\Delta h \to 0$, eq. (2.26.4) becomes, $E_{1t} = E_{2t}$ ...(2.26.5)

4. Thus the tangential components of $\vec{E}$ are the same on the two sides of the boundary. Here $\vec{E}_t$ undergoes no change on the boundary and is said to be continuous across the boundary.
5. Since, $\vec{D} = \epsilon\vec{E} = \vec{D}_t + \vec{D}_n$

   Therefore, eq. (2.26.4) can be written as,

$$\frac{D_{1t}}{\epsilon_1} = E_{1t} = E_{2t} = \frac{D_{2t}}{\epsilon_2}$$

or

$$\frac{D_{1t}}{\epsilon_1} = \frac{D_{2t}}{\epsilon_2} \quad ...(2.26.6)$$

Here D_t undergoes some change across the interface. Thus it is said to be discontinuous across the interface.

6. Applying $\oint_S \vec{D}.d\vec{S} = Q_{enc}$ to the cylindrical Gaussian surface of Fig. 2.26.1, we get

$$\Delta Q = \rho_S\,\Delta S = D_{1n}\,\Delta S - D_{2n}\,\Delta S$$

$$\therefore \quad D_{1n} - D_{2n} = \rho_S \quad ...(2.26.7)$$

7. If no free charges exists at the interface *i.e.,* $\rho_S = 0$ then

$$D_{1n} = D_{2n} \quad ...(2.26.8)$$

or

$$\epsilon_1 E_{1n} = \epsilon_2 E_{2n}$$

8. Let us consider $\vec{D}_1$ or $\vec{E}_1$ and $\vec{D}_2$ or $\vec{E}_2$ making angle θ_1 and θ_2 with the normal to the interface as shown in Fig. 2.26.2.

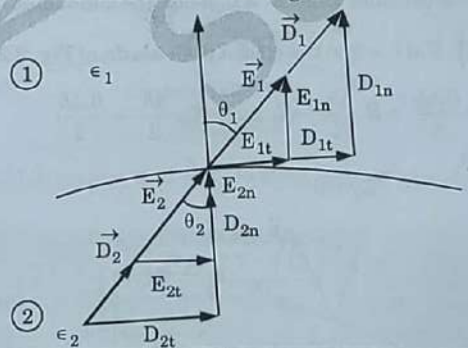


Fig. 2.26.2. Refraction of $\vec{D}$ or $\vec{E}$ at a dielectric-dielectric boundary.

9. From Fig. 2.26.2,

$$E_1 \sin\theta_1 = E_{1t} = E_{2t} = E_2 \sin\theta_2$$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2 \quad ...(2.26.9)$$

Similarly, $\epsilon_1 E_1 \cos\theta_1 = \epsilon_2 E_2 \cos\theta_2 \quad ...(2.26.10)$

Dividing eq. (2.26.9) by (2.26.10), we get

$$\frac{\tan\theta_1}{\epsilon_1} = \frac{\tan\theta_2}{\epsilon_2} \quad ...(2.26.11)$$

Since, $\epsilon_1 = \epsilon_0\epsilon_{r1}$ and $\epsilon_2 = \epsilon_0\epsilon_{r2}$, eq. (2.26.11) becomes

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}$$

10. Thus, an interface between two dielectrics produces binding of flux lines as a result of unequal polarization charges that accumulate on the opposite side of the interface.

Que 2.27. Discuss the concept of conductor-dielectric boundary condition and conductor-free space boundary condition.

OR

Derive and explain boundary conditions for static electric fields.

AKTU 2019-20, Marks 10

Answer

A. **Boundary conditions :** Refer Q. 2.25, Page 2–28B, Unit-2.

B. **Dielectric-dielectric boundary condition :** Refer Q. 2.26, Page 2–29B, Unit-2.

C. **Conductor-dielectric boundary condition :**

1. In order to determine the boundary condition for a conductor-dielectric interface, we consider that $\vec{E} = 0$ inside the conductor.
2. Applying $\oint \vec{E}.\vec{dl} = 0$ to the closed path *abcda* of Fig. 2.27.1 gives

$$0 = 0.\Delta w + \frac{0.\Delta h}{2} + E.\frac{\Delta h}{2} - E_t.\Delta w - E_n.\frac{\Delta h}{2} - \frac{0.\Delta h}{2} \quad ...(2.27.1)$$

As, $\Delta h \to 0$,

$$\therefore \quad E_t = 0 \quad ...(2.27.2)$$

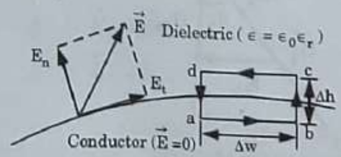


Fig. 2.27.1.

3. Similarly, applying $\oint \vec{D}.\vec{dS} = Q_{enc}$ to the cylindrical Gaussian surface as shown is Fig. 2.27.2 gives

$$\Delta Q = D_n.\Delta S - 0.\Delta S \quad ...(2.27.3)$$

because $\vec{D} = \epsilon\vec{E} = 0$ inside the conductor.

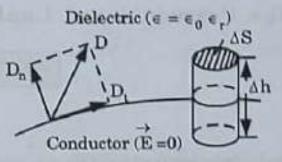


Fig. 2.27.2.

4. Eq. (2.27.3) may be written as

$$D_n = \frac{\Delta Q}{\Delta S} = \rho_S$$

or $\quad D_n = \rho_S$

D. **Conductor-free space boundary conditions :**

1. The conductor-free space boundary conditions are represented in Fig. 2.27.3.
2. As we know, $\quad D_t = \epsilon_0 \epsilon_r E_t = 0$ and $D_n = \epsilon_0 \epsilon_r E_n = \rho_S$

By replacing ϵ_r by 1, the boundary conditions at the interface between a conductor and free space can be obtained

$$D_t = \epsilon_0 E_t = 0$$

$$D_n = \epsilon_0 E_n = \rho_S \quad ...(2.27.4)$$

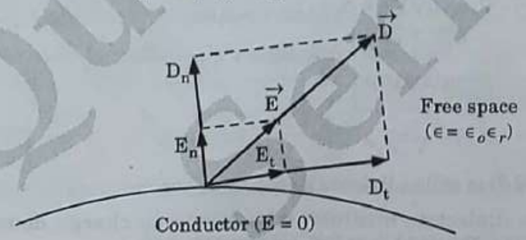


Fig. 2.27.3.

3. The eq. (2.27.3) implies that $\vec{E}$ field must approach a conducting surface normally.

PART-10

Electrostatic Boundary Value Problems : Poisson's and Laplace's Equations, Methods of Images.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.28. **Derive the Poisson's and Laplace equation in all coordinate systems.**

AKTU 2021-22, Marks 10

Answer

A. Poisson's and Laplace's equation :

1. Poisson's and Laplace's equations are used to solve the boundary value problems. Poisson's equation can be derived from the Gauss's law in the point form.
2. Gauss's law in the point form is,
$$\nabla \cdot \vec{D} = \rho_v$$
3. We know that, $\vec{D} = \epsilon \vec{E}$
$$\therefore \quad \nabla \cdot \epsilon \vec{E} = \rho_v \quad \text{...(2.28.1)}$$
4. Now, from the gradient relationship,
$$\vec{E} = -\nabla V \quad \text{...(2.28.2)}$$
5. Substituting eq. (2.28.2) in (2.28.1),
$$\nabla \cdot \epsilon(-\nabla V) = \rho_v$$
$$-\epsilon[\nabla . \nabla V] = \rho_v$$
$$\therefore \quad \nabla \cdot \nabla V = \frac{-\rho_v}{\epsilon}$$
$$\nabla^2 V = \frac{-\rho_v}{\epsilon} \quad \text{...(2.28.3)}$$
Eq. (2.28.3) is called Poisson's equation.
6. Now, in dielectric medium, where volume charge density is zero ($\rho_v = 0$), the Poisson's equation takes the form,
$$\nabla^2 V = 0 \quad \text{(for charge free region)}$$
This is special case of Poisson's equation and is called as Laplace's equation.

B. Poisson's equation in different coordinates system :

1. Cartesian coordinates system,
$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\rho/\varepsilon_0$$

2. Cylindrical coordinates system,
$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = -\rho/\varepsilon_0$$
3. Spherical coordinates system,
$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = -\rho/\varepsilon_0$$

C. Laplace's equation in different coordinate systems :

1. Cartesian coordinates system,
$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$
2. Cylindrical coordinates system,
$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$
3. Spherical coordinates system,
$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$$

Que 2.29. **Calculate the capacitance formed by two back to back cones separated by infinitely small distance.**

AKTU 2017-18, Marks 07

Answer

1. Consider a capacitor is formed by two back to back cones.
Let 2θ = Angle of cones
D = Adjustable distance between apexes of cones
h = Height of inside cone A from its apex to the base of its effective meshed surface.

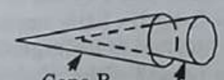


Fig. 2.29.1. Right cone continuously adjustable capacitor.

2. The potential gradient is perpendicular to the two metal surfaces nearly everywhere in the space between them.

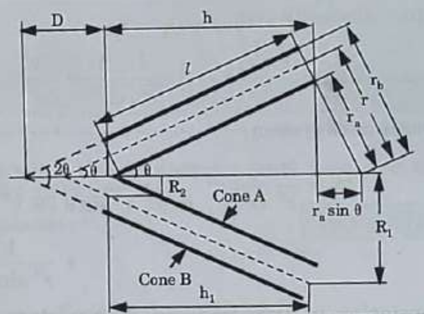


Fig. 2.29.2. Cross-sectional dimensions of conical capacitor.

3. Since the conical conducting surfaces are parallel and coaxial, it follows that the lines of force in the dielectric space (assumed to be air) are everywhere normal to both surfaces.

$$E.\eta = E = -\frac{dr}{d\eta} = \text{Constant} \quad ...(2.29.1)$$

where E = Potential gradient or the electric field intensity
η = Unit vector normal to the surfaces.

4. E is constant over either one of the two conducting surfaces as well. So,

$$E\eta = K\sigma \quad ...(2.29.2)$$

where σ = Surface charge density
K = Constant.

5. Referring to Fig. 2.29.2 and taking the surface S_f, of any other coaxial frustum of half-angle θ, located between the two conical electrodes and applying Gauss law to this surface, we have

$$\int_s D.dS = \epsilon ES_f = Q \quad ...(2.29.3)$$

where the dielectric constant of the interelectrode space is ϵ.

6. The surface of the frustum of altitude h_1 and base radii R_1 and R_2 is

$$S_f = \pi(R_1 + R_2)\sqrt{h_1^2 + (R_1 - R_2)^2}$$

7. In order to express capacitance values in terms of D and h, we have from the geometry of the Fig. 2.29.2 :

$$R_1 = r\cos\theta,\ h_1 = l\cos\theta,\ R_2 = r\cos\theta - l\sin\theta$$

and
$$S_f = \pi(2r\cos\theta - l\sin\theta)\sqrt{l^2\cos^2\theta + l^2\sin^2\theta}$$
$$= \pi l(2r\cos\theta - l\sin\theta) \quad ...(2.29.4)$$

where r = Length of the normal from the axis to this surface at its larger base
l = Length of the meshed conical surfaces.

8. Substituting eq. (2.29.4) into eq. (2.29.3)

$$Q = \epsilon\pi lE(2r\cos\theta - l\sin\theta)$$

and
$$E = Ql[\epsilon\pi l(2r\cos\theta - l\sin\theta)] \quad ...(2.29.5)$$

9. Because of conical symmetry, the potential at the surface of this frustum, as well as of all others, depends only on r; therefore, the potential difference V, between frustum A and B is

$$V = -\int_{r_b}^{r_a} Edr = -\frac{Q}{\epsilon\pi l}\int_{r_b}^{r_a}\frac{dr}{(2r\cos\theta - l\sin\theta)}$$
$$= \frac{-Q}{2\epsilon\pi l\cos\theta}\ln(2r\cos\theta - l\sin\theta)\Big|_{r_b}^{r_a}$$
$$= \frac{Q}{2\epsilon\pi l\cos\theta}\ln\frac{(2r_b\cos\theta - l\sin\theta)}{(2r_a\cos\theta - l\sin\theta)} \quad ...(2.29.6)$$

10. Rearranging to obtain the capacitance we have

$$C = \frac{Q}{V} = \frac{2\pi\epsilon l\cos\theta}{\ln\dfrac{2r_b\cos\theta - l\sin\theta}{2r_a\cos\theta - l\sin\theta}}$$

11. Expressing r_a and r_b in terms of h and D, we have

$$C = \frac{2\pi\epsilon l\cos\theta}{\ln\left[1 + \dfrac{2D\cos^2\theta}{2h - l\cos\theta}\right]}$$

12. When the internal electrode is a cone, it may be considered a cone having a infinitely small separation with outer cone and an altitude $h = l\cos\theta$.

Then
$$C = \frac{2\pi\epsilon h}{\ln[1 + 2(D/h)\cos^2\theta]}\ \mu F \quad ...(2.29.7)$$

Que 2.30. **Derive an expression for capacitance of a spherical shaped capacitor.**

AKTU 2019-20, Marks 10

Answer

1. Fig. 2.30.1 shows a spherical capacitor formed by two concentric spherical shells A and B of radii a and b charged with $+Q$ and $-Q$ charges respectively.
2. These charges attract each other, and spread out uniformly on the outer surface of the inner shell and the inner surface of the outer shell.
3. They produce an electric field $\vec{E}$ between the two shells, radially outwards.
4. Consider a Gaussian surface which is a concentric spherical surface of radius r. The electric flux through it is

$$\phi = \oint_s \vec{E}\cdot\vec{dS} = \oint_s E\,dS \quad ...(2.30.1)$$

$$[\because\ (E.dS = E\,dS\cos 0 = E\,dS)]$$

or
$$\Phi = E\oint_S dS \qquad (\because E \text{ is constant})$$

$$= E(4\pi r^2) \qquad \left(\because \oint_S dS = 4\pi r^2\right) \qquad ...(2.30.2)$$

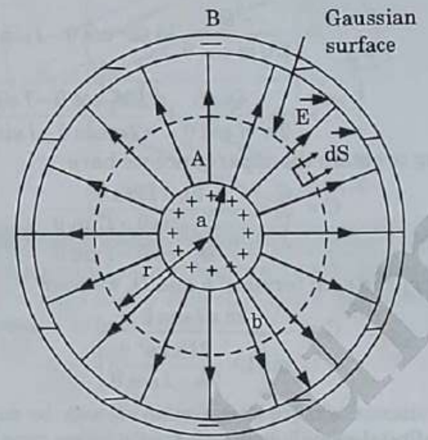


Fig. 2.30.1. Spherical capacitor formed by two concentric shells.

5. According to Gauss's law, the electric flux Φ must be equal to $\frac{1}{\epsilon_0}$ times the charge contained within the surface

$\therefore$
$$\Phi = E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

or
$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2} \qquad ...(2.30.3)$$

6. The potential difference V between the spheres A and B is given by

$$V = V_A - V_B = -\int_b^a E.\vec{dl} \qquad ...(2.30.4)$$

where $\vec{dl}$ is the differential vector displacement along a path from B to A. $\vec{E}$ is radially outward while $\vec{dl}$ (along the direction of motion) is inward and the angle between $\vec{E}$ and $\vec{dl}$ is 180°.

$$\vec{E}.\vec{dl} = Edl \cos 180° = -E\,dl$$

Also
$$dl = -dr$$

So,
$$\vec{E}.\vec{dl} = Edr \qquad ...(2.30.5)$$

7. Put the value of eq. (2.30.5) in eq. (2.30.4) then we get,

$$V = -\int_b^a E\,dr \qquad ...(2.30.6)$$

8. Put the value of eq. (2.30.3) in eq. (2.30.6) then we get,

$$V = -\frac{Q}{4\pi\epsilon_0}\int_b^a \frac{dr}{r^2} = -\frac{Q}{4\pi\epsilon_0}\left(-\frac{1}{r}\right)_b^a$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{b-a}{ab} \qquad ...(2.30.7)$$

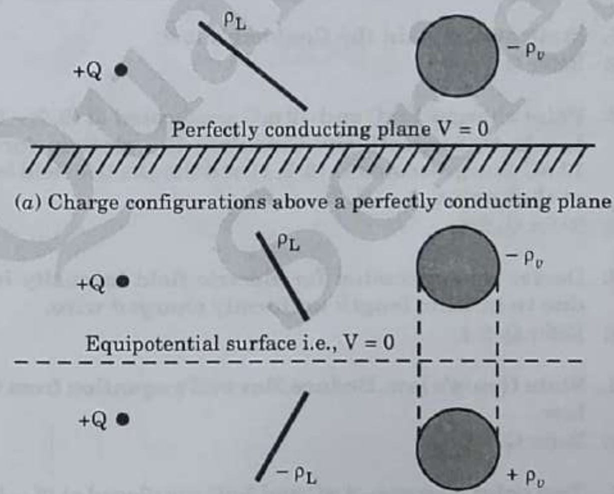
9. But the capacitance of capacitor is $C = \frac{Q}{V}$, so put the value V from eq. (2.30.7) then we get,

$$C = \frac{Q}{V} = 4\pi\epsilon_0 \frac{ab}{b-a}$$

Que 2.31. **Explain methods of images.**

Answer

1. Methods of images helps us to find V, $\vec{E}$, $\vec{D}$ and ρ_S due to charges in the presence of conducting surfaces which are equipotential, without solving Poisson's or Laplace's equation.



(a) Charge configurations above a perfectly conducting plane

(b) Image configuration with the conducting plane replaced by equipotential surface.

Fig. 2.31.1. Image system.

2. Let the plane, exists midway between the two charges, is a zero potential infinite plane. The conductor is an equipotential surface at a potential

   $V = 0$ and $\vec{E}$ is only normal to the surface.
3. If there is a charge above a conducting plane same fields at the points above the plane can be obtained by replacing conducting plane by equipotential surface and an image of the charge at a symmetrical location below the plane.
4. To apply the method of images, following conditions must be satisfied :

i. The image charges must be located in the conducting region.

ii. The large image charges must be located such that on the conducting surface the potential is zero or constant.

5. Here, the first condition is to satisfy Poisson's equation, while the other to satisfy the boundary conditions.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. State and explain the Coulomb's law.

Ans. Refer Q. 2.1.

Q. 2. Point charges 1 mC and – 2 mC are located at (3, 2, – 1) and (– 1, – 1, 4), respectively. Calculate the electric force on a 10 nC charge located at (0, 3, 1) and the electric field intensity at that point.

Ans. Refer Q. 2.3.

Q. 3. Derive an expression for electric field intensity in space due to infinite length uniformly charged wire.

Ans. Refer Q. 2.4.

Q. 4. State Gauss's law. Deduce Maxwell's equation from Gauss's law.

Ans. Refer Q. 2.7.

Q. 5. Two point charges – 4 nC and 5 nC are placed at (2, – 1, 3) and (0, 4, – 2) respectively, find the potential at (1, 0, 1), assuming zero potential at infinity. Right angle triangle. Find electric forces at the corners of the triangle.

Ans. Refer Q. 2.11.

Q. 6. Derive energy density in electrostatic field.

Ans. Refer Q. 2.15.

Q. 7. Discuss some properties of material.

Ans. Refer Q. 2.18.

Q. 8. Explain the phenomena of polarization and its types.

Ans. Refer Q. 2.21.

Q. 9. Derive and explain continuity equation for electrostatic in detail.

Ans. Refer Q. 2.23.

Q. 10. Derive and explain boundary conditions for static electric fields.

Ans. Refer Q. 2.27.

Q. 11. Derive the Poisson's and Laplace equation in all coordinate systems.

Ans. Refer Q. 2.28.

Q. 12. Derive an expression for capacitance of a spherical shaped capacitor.

Ans. Refer Q. 2.30.

☺☺☺

3 UNIT

Magnetostatics

CONTENTS

PART-1

Magneto-static Fields, Biot-Savart's Law.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.1. What is magnetostatic field ?

Answer

1. A magnetostatic field is produced by a constant current flow (or direct current).
2. This current flow may be due to magnetization currents as in permanent magnets, electron beam currents as in vacuum tubes, or conduction currents as in current-carrying wires.
3. There are two major laws governing magnetostatic fields :
i. Biot-Savart's law
ii. Ampere's circuit law.

Que 3.2. State and explain the Biot-Savart's law.

Answer

1. Biot-Savart's law states that the differential magnetic field intensity dH produced at point P, as shown in Fig. 3.2.1, by the differential current element $I\,dl$ is proportional to the product $I\,dl$ and sine of angle (α) between the element and the line joining point P to the element and is inversely proportional to the square of the distance R between P and the element.

i.e., $$dH \propto \frac{I\,dl \sin\alpha}{R^2}$$

or, $$dH = \frac{kI\,dl \sin\alpha}{R^2} \quad ...(3.2.1)$$

where, $k = 1/4\pi$, is the constant of proportionality.

then, $$dH = \frac{I\,dl \sin\alpha}{4\pi R^2} \quad ...(3.2.2)$$

2. In vector form,

$$d\vec{H} = \frac{I\,d\vec{l} \times \hat{a}_R}{4\pi R^2} = \frac{I\,d\vec{l} \times \vec{R}}{4\pi R^3}$$

where, $R = |\vec{R}|$ and $\hat{a}_R = \frac{\vec{R}}{R}$ and $\vec{dl}$ is shown in Fig. 3.2.1.

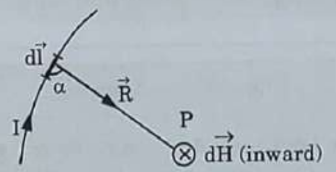


Fig. 3.2.1. Magnetic field $\vec{dH}$ at P due to current element $I\,\vec{dl}$.

3. Biot-Savart's law in terms of distributed current sources is given as :

$$\vec{H} = \int_L \frac{I\,\vec{dl} \times \hat{a}_R}{4\pi R^2} \qquad \text{(Line current)}$$

$$\vec{H} = \int_S \frac{\vec{K}\,dS \times \hat{a}_R}{4\pi R^2} \qquad \text{(Surface current)}$$

$$\vec{H} = \int_v \frac{\vec{J}\,dv \times \hat{a}_R}{4\pi R^2} \qquad \text{(Volume current)}$$

where, $\hat{a}_R$ is a unit vector pointing from the differential element of current to the point of interest, $\vec{K}$ is surface current density and $\vec{J}$ is volume current density.

Que 3.3. Explain Biot Savert's law for magnetic fields. How this concept can be used to determine magnetic field in space due to a close loop current carrying wire ? **AKTU 2019-20, Marks 10**

Answer

i. Biot Savert's law : Refer Q. 3.2, Page 3–2B, Unit-3.

ii. Magnetic field due to a close loop current carrying wire :

1. Fig. 3.3.1 shows a current carrying loop of radius a. P is a point on the axis of the loop distant x from the centre, at which the field is to be determined.
2. Let $\vec{dl}$ be a current element at the loop. Let $\vec{R}$ is the displacement vector from the current element to P. Biot-Savart's law gives the magnetic field at P due to the current element as dl as,

$$d\vec{H} = \frac{1}{4\pi}\frac{I\,\vec{dl} \times \vec{R}}{R^3} \qquad ...(3.3.1)$$

The magnitude of $d\vec{H}$ is,

$$d\vec{H} = \frac{1}{4\pi}\frac{I\,dl\sin 90°}{R^2} = \frac{\mu_0 I\,dl}{4\pi\,R^2} \qquad ...(3.3.2)$$

[$\because$ Angle between $\vec{dl}$ and $\vec{R}$ is 90°]

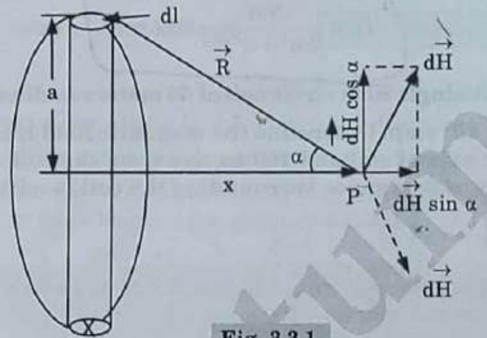


Fig. 3.3.1.

3. Consider another current-element $\vec{dl}$ of the same length dl at the bottom of the coil perpendicular into the page.
4. The magnetic field $d\vec{H}$ due to this element is equal to dH in magnitude.
5. The components of $d\vec{H}$ and $d\vec{H}$, in a direction perpendicular to the axis of the loop, are equal and opposite. Therefore, they cancel each other.
6. The components of $d\vec{H}$ and $d\vec{H}$ along the axis are in the same direction and hence they are added.
7. The magnitude of the field is,

$$H = \int dH \sin\alpha \qquad ...(3.3.3)$$

Putting the value of dH from eq. (3.3.2) in eq. (3.3.3) then we get, Tesla

$$\vec{H} = \frac{I}{4\pi R^2}\int dl \sin\alpha \qquad ...(3.3.4)$$

From Fig. 3.3.1, $\sin\alpha = \frac{a}{R}$

8. Put the value of sin α in eq. (3.3.4) then we get,

$$\therefore \quad H = \frac{Ia}{4\pi R^3}\int dl \qquad ...(3.3.5)$$

But $\int dl = 2\pi a$ (circumference of the coil)

$$R = (a^2 + x^2)^{3/2}$$

Put the value of $\int dl$ and R in eq. (3.3.5) then, we get

$$\therefore \quad H = \frac{Ia^2}{2(a^2 + x^2)^{3/2}}$$

9. If there are N turns in the coil, then

$$H = \frac{Nia^2}{2(a^2 + x^2)^{3/2}} \text{ Wb/ m}^2$$

Que 3.4. **A single turn circle coil of 50 meters in diameter carries current 28 × 10⁴ amp. Determine the magnetic field intensity H at a point on the axis of coil and 100 meters from the coil. The relative permeability of free space surrounding the coil is unity.**

Answer

Given : Diameter = 50 m, *i.e.*, Radius, a = 25 m, $I = 28 \times 10^4$ amp, Distance from center (x) = 100 m, $\mu_0 = 1$
To Find : H at 100 m distance.

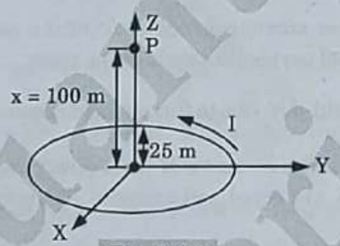


Fig. 3.4.1.

$$H = \frac{Ia^2}{2(a^2 + x^2)^{3/2}} \vec{a}_z$$

$$= \frac{28 \times 10^4 \times (25)^2}{2[(25)^2 + (100)^2]^{3/2}} \vec{a}_z = 79.89 \vec{a}_z \text{ A/m}$$

Que 3.5. **Evaluate magnetic field intensity in space due to current wire.**

AKTU 2018-19, Marks 07

OR

Explain Biot-Savart's Law. Also derive an expression for a magnetic field intensity in space due to an infinite uniform current carrying wire.

AKTU 2018-19, Marks 07

OR

Explain Biot-Savart's law. Find the magnetic field intensity for infinite line current.

AKTU 2021-22, Marks 10

OR

Determine the magnetic flux density B at a distance d meter from an infinite straight wire carrying current I. Also find out when the length of the wire is semi-infinite.

AKTU 2017-18, Marks 10

Answer

A. **Biot-Savart's law :** Refer Q. 3.2, Page 3–2B, Unit-3.

B. **Magnetic field intensity due to current carrying wire :**

1. To determine the field due to a straight current carrying filamentary conductor of finite length AB is shown in Fig. 3.5.1.

$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin\theta}{r^2}$ (1)

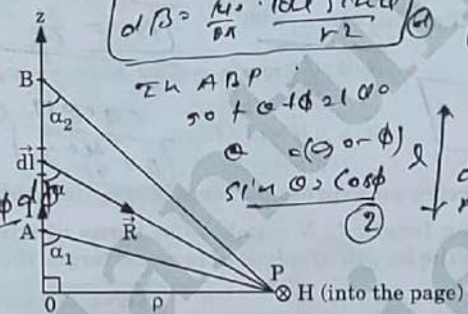


Fig. 3.5.1. Field at point P due to a straight filamentary conductor.

$\rho = \tan\phi$

$\frac{l}{a} = \tan\phi$

$l = a\tan\phi$

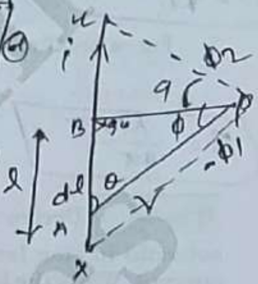
$dl = a\sec^2\phi\, d\phi$ (3)

In ABP

$90 + \theta + \phi = 180$

$\theta = (90 - \phi)$

$\sin\theta = \cos\phi$ (2)



$\cos\phi = \frac{a}{r}$ (4)

$r = \frac{a}{\cos\phi}$

2. Let the conductor is along the z-axis with its upper and lower ends, subtending angles α_2 and α_1 at P, the point at which $\vec{H}$ is to be determined.

3. Here the current flows from point A, where $\alpha = \alpha_1$ to point B, where $\alpha = \alpha_2$.

4. The value of $d\vec{H}$ at P due to an element $\vec{dl}$ at (0, 0, z) is

$$d\vec{H} = \frac{I\,\vec{dl} \times \vec{R}}{4\pi R^3} \quad ...(3.5.1)$$

$\frac{Idl\, a\sec^2\phi\, d\phi \cdot \cos\phi}{\left(\frac{a^2}{\cos^2\phi}\right)}$

$\Rightarrow dB = \frac{\mu_0}{4\pi} \times I \cos\phi\, d\phi$ (5)

but $\quad \vec{dl} = dz\, \hat{a}_z$ and $\vec{R} = \rho\hat{a}_\rho - z\hat{a}_z$

$dB = \frac{\mu_0}{4\pi} \int_{-\phi_1}^{\phi_2} \cos\phi\, d\phi$

so, $\quad \vec{dl} \times \vec{R} = \rho dz\, \hat{a}_\phi$

$$\therefore \quad \vec{H} = \int \frac{I\rho\, dz}{4\pi[\rho^2 + z^2]^{3/2}} \hat{a}_\phi \quad ...(3.5.2)$$

$B \Rightarrow \frac{\mu_0}{4\pi} I [\sin\phi_2 + \sin\phi_1]$

5. Let $z = \rho\cot\alpha$, $dz = -\rho\operatorname{cosec}^2\alpha$, thus eq. (3.5.2) becomes

current direction → According to right thumb rule
⇒ ↑ perpendicular to the plane (I ...

$$\vec{H} = \frac{-I}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \text{ cosec}^2 \alpha \, d\alpha}{\rho^3 \text{ cosec}^3 \alpha} \hat{a}_\phi$$

$$\vec{H} = \frac{-I}{4\pi\rho} \hat{a}_\phi \int_{\alpha_1}^{\alpha_2} \sin \alpha \, d\alpha$$

$$\therefore \quad \vec{H} = \frac{I}{4\pi\rho} (\cos \alpha_2 - \cos \alpha_1) \, \hat{a}_\phi \qquad ...(3.5.3)$$

6. If conductor is infinite in length, then point A is at $(0, 0, -\infty)$ while B is at $(0, 0, \infty)$; $\alpha_1 = 180°$ and $\alpha_2 = 0°$, thus eq. (3.5.3) reduces to

$$\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\phi$$

case I $\theta_1 = \theta_2 = 90$

$$\vec{B} = \mu\vec{H} = \mu I / 2\pi\rho \, \hat{a}_\phi$$

$B = \frac{\mu_0}{4\pi} I [...]$

$\boxed{B = \frac{\mu}{4\pi} 2I}$ $\boxed{B = \frac{\mu}{4\pi} 2I}$

7. For semi-infinite, $\alpha_2 = 0°$, $\alpha_1 = 90°$

$$\vec{H} = \frac{I}{2\pi\rho} (\cos \alpha_2 - \cos \alpha_1) \, \hat{a}_\phi = \frac{I}{4\pi\rho} \hat{a}_\phi$$

$$\vec{B} = \mu\vec{H} = \mu I / 4\pi\rho \, \hat{a}_\phi$$

case II $\phi_1 = 0$ $\phi_2 = 90$

$\boxed{B = \frac{\mu_0 I}{4\pi}}$ $\boxed{B = \frac{\mu_0}{4\pi} I}$

Que 3.6. **Derive an expression for a magnetic field intensity in solenoid having length *L*, *N* numbers of turns of wire carrying *I* current. While the length of solenoid is much larger then it's radius.**

AKTU 2018-19, Marks 07

OR

A solenoid of length '*l*' and radius '*a*' consists of *N* turns of wire carrying current *I*. Show that at point P along its axis

$$\vec{H} = \frac{nI}{2} (\cos \theta_2 - \cos \theta_1) \, \hat{a}_z$$

where, $n = N/l$, θ_1 and θ_2 are the angles subtended at *P* by the end turns as given in Fig. 3.6.1. Also show that if $l >> a$, at the centre of the solenoid,

$$\vec{H} = nI \, \hat{a}_z$$

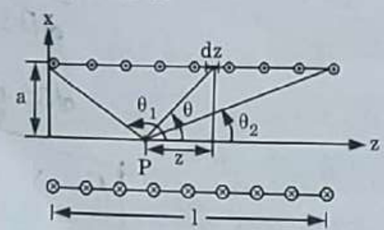


Fig. 3.6.1. Cross-section of a solenoid.

Answer

1. As the solenoid consists of circular loops, therefore the contribution to the magnetic field *H* at P by an element of the solenoid of length *dz* is

$$dH_z = \frac{I \, dl \, a^2}{2[a^2 + z^2]^{3/2}} = \frac{Ia^2 \, n \, dz}{2[a^2 + z^2]^{3/2}} \qquad ...(3.6.1)$$

where, $dl = n \, dz = (N/l) \, dz$.

2. From Fig. 3.6.1, $\tan \theta = a/z$

$$\therefore \quad dz = -a \text{ cosec}^2 \theta \, d\theta = -\frac{[z^2 + a^2]^{3/2}}{a^2} \sin \theta \, d\theta$$

3. Putting the value of *dz* in eq. (3.6.1), we get

$$dH_z = \frac{-nI}{2} \sin \theta \, d\theta$$

or
$$H_z = -\frac{nI}{2} \int_{\theta_1}^{\theta_2} \sin \theta \, d\theta$$

Thus,
$$\vec{H} = \frac{nI}{2} (\cos \theta_2 - \cos \theta_1) \, \hat{a}_z$$

Substituting $n = N/l$ gives,

$$\vec{H} = \frac{NI}{2l} (\cos \theta_2 - \cos \theta_1) \, \hat{a}_z$$

4. At the center of the solenoid,

$$\cos \theta_2 = \frac{l/2}{[a^2 + l^2/4]^{1/2}} = -\cos \theta_1$$

and
$$\vec{H} = \frac{nIl}{2[a^2 + l^2/4]^{1/2}} \hat{a}_z$$

5. If $l >> a$ or $\theta_2 = 0°$ and $\theta_1 = 180°$

$$\vec{H} = nI \, \hat{a}_z = \frac{NI}{l} \hat{a}_z$$

PART-2

Ampere's Circuit Law-Maxwell's Equation, Applications of Ampere's Law .

Questions-Answers
Long Answer Type and Medium Answer Type Questions

Que 3.7. **What is Ampere's circuit law ?**

OR

State and prove Ampere's circuit law and derive Maxwell's equation for it.

OR

State and derive ampere circuital law. A single turn circle coil of 50 meters in diameter carries current 28×10^4 amp. Determine the magnetic field intensity H at a point on the axis of coil and 100 meters from the coil. The relative permeability of free space surrounding the coil is unity. **AKTU 2017-18, Marks 07**

Answer

A. Ampere's circuit law :

a. Statement :

1. The Ampere's circuit law states that, "The line integral of magnetic field intensity around any closed path is exactly equal to the direct current enclosed by that path".
2. The Ampere's circuit law is given by

$$\boxed{\oint \vec{H} \cdot \vec{dl} = I_{enc}} \quad \text{or} \quad \boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 I} \quad ...(3.7.1)$$

where, $\vec{H}$ = Magnetic field intensity

$\vec{dl}$ = Infinitesimal small (differential) element of wire

I_{enc} = Current enclosed.

b. Proof :

1. Fig. 3.7.1 shows a long straight conductor carrying a current I in the direction shown. Biot-Savart's law gives the magnitude of the magnetic field intensity at a distance R from it.

$$H = I/2\pi R$$

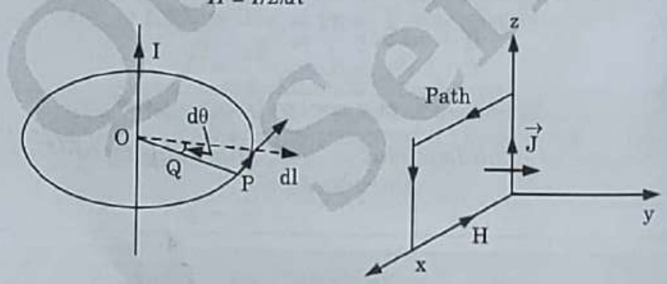


Fig. 3.7.1. Ampere's circuital law.

2. Its direction is given by the tangent to a circle of radius R centred on the current carrying wire.
3. The magnetic field $\vec{H}$ is constant in magnitude at all points of this circle and its direction is parallel to the circle element $\vec{dl}$.

$$\therefore \quad \oint \vec{H}.\vec{dl} = \oint \frac{I}{2\pi R} R d\theta = \frac{I}{2\pi} \oint d\theta$$

$$= \frac{I}{2\pi} . 2\pi$$

$$\oint \vec{H}.\vec{dl} = I$$

4. Thus, the line integral. $\oint \vec{H}.\vec{dl}$ is equal to the current through the area bounded by the circles. It is Ampere's law.

B. Maxwell equation from Ampere's law :

1. Applying Stoke's theorem in eq. (3.7.1), we get

$$I_{enc} = \oint \vec{H} \cdot \vec{dl} = \int_s (\nabla \times \vec{H}) \cdot d\vec{S} \quad ...(3.7.2)$$

As, $$I_{enc} = \int_s \vec{J} \cdot d\vec{S} \quad ...(3.7.3)$$

2. Comparing the surface integral in eq. (3.7.2) and (3.7.3) clearly indicates that

$$\boxed{\nabla \times \vec{H} = \vec{J}} \quad ...(3.7.4)$$

3. Eq. (3.7.4) is ~~the third Maxwell~~'s equation.

C. Numerical : Refer Q. 3.4, Page 3–5B, Unit-3.

Que 3.8. Explain the applications of Ampere's law for infinite (*i*) line current, (*ii*) sheet of current (*iii*) long coaxial transmission line.

OR

Derive an expression for magnetic field intensity in space due to current sheet having current density Ka_x A/m.

AKTU 2019-20, Marks 10

OR

Derive an expression for magnetic field of a coaxial cable.

AKTU 2019-20, Marks 10

OR

Explain the Ampere circuital law. Derive two applications of ampere circuital law. Also, derive modified Maxwell's equations.

AKTU 2021-22, Marks 10

Answer

A. Applications of ampere circuited law :

i. Infinite line current :

1. Consider an infinitely long filamentary current I along the z-axis as shown in the Fig. 3.8.1.

2. To determine $\vec{H}$ at point P, consider a closed path passing through point P. This is the path on which Ampere's law is applied and it is known as Amperian path.
3. Since this path encloses the whole current I, then according to Ampere's law,

$$I = \int H_\phi\, \hat{a}_\phi \cdot \rho\, d\phi\, \hat{a}_\phi$$

$$I = H_\phi \int \rho\, d\phi = H_\phi\, 2\pi\rho$$

$$\boxed{\vec{H} = \frac{I}{2\pi\rho}\,\hat{a}_\phi}$$ Vector form

$\oint H \cdot dl = I$

$H \oint dl = I$

$H\,(2\pi r) = I$

$\boxed{H = \frac{I}{2\pi r}}$

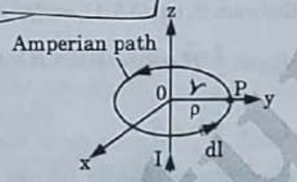


Fig. 3.8.1.

ii. Infinite sheet of current :

1. Consider an infinite current sheet in the $z = 0$ plane.
2. If the sheet has a uniform current density $\vec{K} = K_y\, \hat{a}_y$ A/m as shown in Fig. 3.8.2, applying Ampere's law to the rectangular closed path 1-2-3-4-1 gives

$$\oint \vec{H} \cdot \vec{dl} = I_{enc} = K_y\, b \quad ...(3.8.1)$$

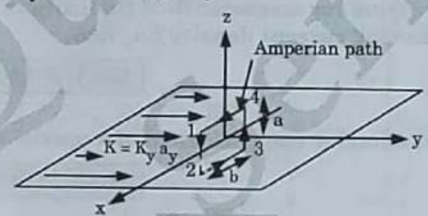


Fig. 3.8.2.

3. As

$$\vec{H} = \begin{cases} H_0 \hat{a}_x \;; & z > 0 \\ -H_0 \hat{a}_x \;; & z < 0 \end{cases} \quad ...(3.8.2)$$

Evaluating the line integral of $\vec{H}$ in eq. (3.8.2) along the closed path as shown in Fig. 3.8.2, gives

$$\oint \vec{H} \cdot \vec{dl} = \left(\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1\right) \vec{H} \cdot \vec{dl}$$

$$= 0\,(-a) + (-H_0)(-b) + 0\,(a) + (H_0)(b)$$

$$\oint \vec{H} \cdot \vec{dl} = 2H_0 b \quad ...(3.8.3)$$

4. From eq. (3.8.1) and (3.8.3), we get

$$K_y\, b = 2H_0 b$$

$$\therefore \quad H_0 = \frac{1}{2} K_y.$$

5. Substituting the value of H_0 in eq. (3.8.2), we get

$$\vec{H} = \begin{cases} \frac{1}{2} K_y\, \hat{a}_x \;; & z > 0 \\ \frac{-1}{2} K_y\, \hat{a}_x \;; & z < 0 \end{cases} \quad ...(3.8.4)$$

6. For an infinite sheet of current density $\vec{K}$ A/m

$$\vec{H} = \frac{1}{2} \vec{K} \times \hat{a}_n \quad ...(3.8.5)$$

where, $\hat{a}_n$ is unit normal vector directed from the current sheet to the point of interest.

iii. Infinitely long coaxial transmission line :

1. Consider an infinitely long transmission line consisting of two concentric cylinders having their axis along the z-axis as shown in the Fig. 3.8.3.
2. The inner conductor has radius a and carries current I, while the outer conductor has inner radius b and thickness t and carries return current $-I$.
3. Since, the current distribution is symmetrical, we apply Ampere's law along Amperian path for each of four possible regions : $0 \le \rho \le a$, $a \le \rho \le b$, $b \le \rho \le b + t$ and $\rho \ge b + t$.
4. For region $0 \le \rho \le a$, we apply Ampere's law to path L_1, *i.e.*,

$$\oint_{L_1} \vec{H} \cdot \vec{dl} = I_{enc} = \int \vec{J} \cdot \vec{dS} \quad ...(3.8.6)$$

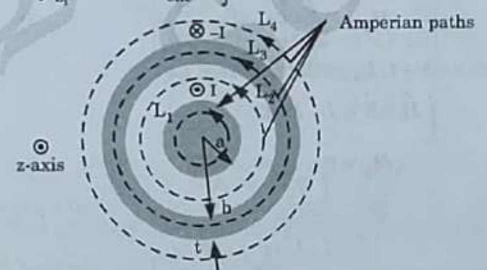


Fig. 3.8.3. Cross-section of the transmission line.

5. As the current is uniformly distributed over the cross-section,

$$\vec{J} = \frac{I}{\pi a^2}\,\hat{a}_z, \quad d\vec{S} = \rho\, d\phi\, d\rho\, \hat{a}_z$$

$$I_{enc} = \int \vec{J}\cdot d\vec{S} = \frac{I}{\pi a^2}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{a} \rho\, d\phi\, d\rho = \frac{I}{\pi a^2}\pi\rho^2 = \frac{I\rho^2}{a^2}$$

6. Thus eq. (3.8.6) becomes, $H_\phi \int_{L_1} dl = H_\phi\, 2\pi\rho = \dfrac{I\rho^2}{a^2}$

or, $$H_\phi = \frac{I\rho}{2\pi a^2} \quad \text{...(3.8.7)}$$

7. For region $a \le \rho \le b$, path L_2 is used as an Amperian path

$$\oint_{L_2} \vec{H}\cdot d\vec{l} = I_{enc} = I$$

$$H_\phi\, 2\pi\rho = I$$

or, $$H_\phi = \frac{I}{2\pi\rho} \quad \text{...(3.8.8)}$$

8. For region $b \le \rho \le b + t$, path L_3 is used

$\therefore$ $$\oint_{L_3} \vec{H}\cdot d\vec{l} = H_\phi\, 2\pi\rho = I_{enc} \quad \text{...(3.8.9)}$$

where, $$I_{enc} = I + \int \vec{J}\cdot d\vec{S}$$

9. Here, $\vec{J}$ is the current density of outer conductor and is along $-\hat{a}_z$, *i.e.*,

$$\vec{J} = -\frac{I}{\pi[(b+t)^2 - b^2]}\hat{a}_z$$

$\therefore$ $$I_{enc} = I - \frac{I}{\pi[(b+t)^2 - b^2]}\int_{\phi=0}^{2\pi}\int_{\rho=b}^{\rho} \rho\, d\rho\, d\phi$$

$$I_{enc} = I\left[1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right]$$

10. Substituting the value of I_{enc} in eq. (3.8.9), we get

$$H_\phi = \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right]$$

For region $\rho \ge b + t$, L_4 path is chosen, *i.e.*,

$$\oint_{L_4} \vec{H}\cdot d\vec{l} = I - I = 0$$

or $$H_\phi = 0$$

So, $$\vec{H} = \begin{cases} \dfrac{I\rho}{2\pi a^2}\hat{a}_\phi & ;0 \le \rho \le a \\ \dfrac{I}{2\pi\rho}\hat{a}_\phi & ;a \le \rho \le b \\ \dfrac{I}{2\pi\rho}\left[1 - \dfrac{\rho^2 - b^2}{t^2 + 2bt}\right]\hat{a}_\phi & ;b \le \rho \le b+t \\ 0 & ;\rho \ge b+t \end{cases}$$

B. Ampere Circuital law and Maxwell equation : Refer Q. 3.7, Page 3–8B, Unit-3.

Que 3.9. **Derive an expression for magnetic field intensity due to long hollow conducting cylinder.**

Answer

1. Consider an infinitely long cylinder of inner radius 'a' and outer radius 'b'. Let 'I' be the current.
2. Suppose that the current is uniformly distributed over whole cross-section of cylinder.

Fig. 3.9.1.

3. For $\rho < a$, $\oint \vec{H}_\phi \cdot d\vec{l} = I_{enc} = 0 \Rightarrow H_\phi = 0$
4. For $a < \rho < b$, $H_\phi\, 2\pi\rho = \dfrac{I\pi(\rho^2 - a^2)}{\pi(b^2 - a^2)}$

$$H_\phi = \frac{I(\rho^2 - a^2)}{2\pi\rho(b^2 - a^2)}$$

5. For $\rho > b$, $H_\phi\, 2\pi\rho = I$

$$H_\phi = \frac{I}{2\pi\rho}$$

6. So, $$H_\phi = \begin{cases} 0 & ;\ \rho < a \\ \dfrac{I}{2\pi\rho}\dfrac{(\rho^2 - a^2)}{(b^2 - a^2)} & ;\ a < \rho < b \\ \dfrac{I}{2\pi\rho} & ;\ \rho > b \end{cases}$$

PART-3

Magnetic Flux Density-Maxwell's Equation, Maxwell's Equation for Static Fields.

Questions-Answers
Long Answer Type and Medium Answer Type Questions

Que 3.10. **What do you understand by magnetic flux density ?**

OR

Derive the Maxwell's equation for flux density.

Answer

1. The magnetic flux density $\vec{B}$ is related to magnetic field intensity $\vec{H}$ as

$$\vec{B} = \mu_0 \vec{H} \quad ...(3.10.1)$$

where, μ_0 is constant known, as permeability of free space.

i.e., $\mu_0 = 4\pi \times 10^{-7}$ H/m

2. The magnetic flux through a surface S is given by

$$\psi = \int_S \vec{B} \cdot d\vec{S} \quad ...(3.10.2)$$

where, ψ is magnetic flux in webers (Wb), $\vec{B}$ is magnetic flux density in Wb/m^2 or tesla (T).

3. A magnetic flux line is a path to which $\vec{B}$ is tangential at every point on the line.
4. As an isolated magnetic charge does not exist thus the total flux through a closed surface in a magnetic field must be zero, *i.e.,*

$$\int_S \vec{B} \cdot d\vec{S} = 0 \quad ...(3.10.3)$$

5. Eq. (3.10.3) is referred to as the law of conservation of magnetic flux or Gauss's law for magnetostatic fields.
6. Applying divergence theorem to eq. (3.10.3), we get

$$\oint_S \vec{B} \cdot d\vec{S} = \oint_v \nabla \cdot \vec{B}\, dv = 0$$

$$\nabla \cdot \vec{B} = 0 \quad ...(3.10.4)$$

7. Eq. (3.10.4) is the fourth Maxwell's equation.

Que 3.11. **Discuss Maxwell's equations for static field.**

Answer

A. Maxwell's first equation :

1. It is derived from Gauss's law for electrostatic fields.
2. According to Gauss's law

$$Q_{enc} = \oint_S \vec{D} \cdot d\vec{S} \quad ...(3.11.1)$$

In terms of volume charge density ρ_v

$$\oint_S \vec{D} \cdot d\vec{S} = \int_v \rho_v\, dv \text{ (Integral form)} \quad ...(3.11.2)$$

3. By applying divergence theorem to eq. (3.11.2)

$$\oint_S \vec{D} \cdot d\vec{S} = \int_v (\nabla \cdot \vec{D})\, dv \quad ...(3.11.3)$$

4. Comparing the two volume integrals in eq. (3.11.2) and (3.11.3) results in

$$\nabla \cdot \vec{D} = \rho_v \text{ (Differential or point form)} \quad ...(3.11.4)$$

5. Eq. (3.11.2) is the integral form, and eq. (3.11.4) is the differential (or point) form of Maxwell's equation.

B. Maxwell's second equation :

1. It is derived from the conservative nature of electrostatic field. A vector field whose line integral does not depend on the path of integration is called conservative vector field.

$$\oint \vec{E} \cdot \vec{dl} = 0 \text{ (Integral form)} \quad ...(3.11.5)$$

This implies that no net work is done in moving a charge along a closed path in an electrostatic field.

2. Applying Stoke's theorem to eq. (3.11.5) gives

$$\oint_L \vec{E} \cdot \vec{dl} = \int_S (\nabla \times \vec{E}) \cdot d\vec{S} = 0$$

or $$\nabla \times \vec{E} = 0 \text{ (Differential or point form)} \quad ...(3.11.6)$$

3. Any vector field that satisfies eq. (3.11.5) or (3.11.6) is said to be conservative, or irrotational.
4. Eq. (3.11.5) is the integral form, and eq. (3.11.6) is the differential (or point) form of Maxwell's equation.

C. Maxwell's third equation :

1. It is derived from Ampere's circuit law, according to this law,

$$\oint \vec{H} \cdot \vec{dl} = I_{enc} = \int_S \vec{J} \cdot d\vec{S} \text{ (Integral form)} \quad ...(3.11.7)$$

2. Applying Stoke's theorem in eq. (3.11.7) gives

$$\oint \vec{H} \cdot \vec{dl} = \int_S (\nabla \times \vec{H}) \cdot d\vec{S} = \int_S \vec{J} \cdot d\vec{S} \quad ...(3.11.8)$$

3. Comparing the surface integrals in eq. (3.11.8) gives

$$\nabla \times \vec{H} = \vec{J} \text{ (Differential or point form)} \quad ...(3.11.9)$$

4. Eq. (3.11.7) is the integral form and eq. (3.11.9) is the differential (or point) form of Maxwell's equation.

D. Maxwell's fourth equation :

1. It is derived from Gauss's law for magnetostatic fields, according to this law

$$\oint \vec{B} \cdot d\vec{S} = 0 \text{ (Integral form)} \quad ...(3.11.10)$$

2. This is because it is not possible to have isolated magnetic poles (or magnetic charges). Eq. (3.11.10) is referred to as the law of conservation of magnetic flux.
3. Applying divergence theorem to eq. (3.11.10) gives

$$\oint_S \vec{B} \cdot d\vec{S} = \int_S \nabla \cdot \vec{B}\, dv = 0$$

or, $$\nabla \cdot \vec{B} = 0 \text{ (Differential or point form)} \quad ...(3.11.11)$$

4. Eq. (3.11.10) is the integral form and eq. (3.11.11) is the differential (or point) form of Maxwell's equation.

PART-4

Magnetic Scalar and Vector Potential.

Questions-Answers
Long Answer Type and Medium Answer Type Questions

Que 3.12. Explain the terms magnetic scalar and vector potential.

OR

Explain convection and conduction currents. Derive mathematical equations also. Also derive the magnetic vector potential.

AKTU 2021-22, Marks 10

Answer

A. Magnetic scalar potential :

1. The magnetic scalar potential is written as V_m and is related to $\vec{H}$ as

$$\vec{H} = -\nabla V_m \quad ...(3.12.1)$$

2. The curl of the magnetic field is given as

$$\nabla \times \vec{H} = \vec{J} = \nabla \times (-\nabla V_m) \quad ...(3.12.2)$$

3. As the curl of the gradient of any scalar identity is zero, therefore it is observed that if magnetic field intensity is to be defined as the gradient of magnetic scalar potential then current density is zero throughout the region in which scalar magnetic potential is defined. Thus,

$$\vec{H} = -\nabla V_m \text{ (for } \vec{J} = 0)$$

4. Since, V_m satisfies the Laplace's equation just as V does for electrostatic fields.

$$\therefore \quad \nabla^2 V_m = 0$$

5. The characteristics of scalar magnetic potential (V_m) are :
   i. The negative gradient of V_m gives $\vec{H}$, *i.e.*, $\vec{H} = -\nabla V_m$
   ii. It exists where $\vec{J} = 0$
   iii. It is directly defined as $V_m = \int_A^B \vec{H} \cdot \vec{dl}$

B. Magnetic vector potential :

1. Vector magnetic potential exists in regions where $\vec{J}$ is present. It is defined in such a way that its curl gives the magnetic flux density, *i.e.*,

$$\vec{B} = \nabla \times \vec{A}$$

where, $\vec{A}$ = Vector magnetic potential (Wb/m).

2. $\vec{A}$ can also be defined for line current, for surface current and for volume current as ;

$$\vec{A} = \int_L \frac{\mu_0 I\, \vec{dl}}{4\pi R} \quad \text{(for line current)}$$

$$\vec{A} = \int_S \frac{\mu_0 \vec{K}\, d\vec{S}}{4\pi R} \quad \text{(for surface current)}$$

$$\vec{A} = \int_v \frac{\mu_0 \vec{J}\, dv}{4\pi R} \quad \text{(for volume current)}$$

3. The characteristics of vector magnetic potential are :
   i. It exists even when $\vec{J}$ is present.
   ii. Vector magnetic potential $\vec{A}$ has applications to obtain radiation characteristics of antennas, apertures and also obtain radiation leakages from transmission line , waveguides and microwave ovens.
   iii. $\vec{A}$ is used to find near and far fields of antennas.

C. Convection and conduction current : Refer Q. 2.19, Page 2–22B, Unit-2.

Que 3.13. Given the magnetic vector potential $A = -\rho^2 / 4\hat{a}_z$ Wb/m. Calculate the total magnetic flux crossing the surface $\phi = \pi/2$, $1 \le \rho \le 2$ m, $0 \le z \le 5$ m.

Answer

1. Given, $\vec{A} = -\rho^2/4\ \hat{a}_z$

2. $$\vec{B} = \nabla \times \vec{A} = -\frac{\partial A_z}{\partial \rho}\hat{a}_\phi = \frac{\rho}{2}\hat{a}_\phi$$

$$d\vec{S} = d\rho dz\ \hat{a}_\phi$$

3. $$\psi = \int \vec{B}\cdot d\vec{S} = \frac{1}{2}\int_{z=0}^{5}\int_{\rho=1}^{2} \rho\, d\rho\, dz$$

$$= \frac{1}{4}\rho^2\Big|_1^2 (5) = 15/4 = 3.75 \text{ Wb}$$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Explain Biot Savert's law for magnetic fields. How this concept can be used to determine magnetic field in space due to a close loop current carrying wire ?

Ans. Refer Q. 3.3.

Q. 2. Evaluate magnetic field intensity in space due to current wire.

Ans. Refer Q. 3.5.

Q. 3. Derive an expression for a magnetic field intensity in solenoid having length *L*, *N* numbers of turns of wire carrying *I* current. While the length of solenoid is much larger then it's radius.

Ans. Refer Q. 3.6.

Q. 4. Derive an expression for magnetic field of a coaxial cable.

Ans. Refer Q. 3.8.

Q. 5. Discuss Maxwell's equations for static field.

Ans. Refer Q. 3.11.

Q. 6. Explain the terms magnetic scalar and vector potential.

Ans. Refer Q. 3.12.

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P: 70

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UNIT

Magnetic Forces

CONTENTS

PART-1

Materials and Devices, Forces Due to Magnetic Fields.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.1. **Discuss the ways in which force due to magnetic fields can be experienced.**

Answer

There are three ways in which force due to magnetic fields can be experienced :

1. **Force on a charged particle :**

i. The electric force F_e on a stationary or moving electric charge Q in an electric field is given by Coulomb's law and is related to $\vec{E}$ as

$$\vec{F}_e = Q\vec{E} \quad ...(4.1.1)$$

The eq. (4.1.1) indicates that if Q is positive, $\vec{F}_e$ and $\vec{E}$ has the same direction.

ii. The magnetic field can exert force only on a moving charge. It is known that magnetic force $\vec{F}_m$ experienced by charge Q moving with velocity $\vec{u}$ in a magnetic field $\vec{B}$ is

$$\vec{F}_m = Q(\vec{u} \times \vec{B}) \quad ...(4.1.2)$$

The eq. (4.1.2) indicates that $\vec{F}_m$ is perpendicular to both $\vec{u}$ and $\vec{B}$.

iii. For a moving charge Q in presence of both electric and magnetic field, total force on the charge is given by

$$\vec{F} = \vec{F}_e + \vec{F}_m$$

or

$$\vec{F} = Q(\vec{E} + \vec{u} \times \vec{B})$$

iv. This is known as Lorentz force equation.

2. **Force on a current element :**

i. To determine the force on a current element $I\,\vec{dl}$ of a current carrying conductor due to magnetic field $\vec{B}$, we will use fact of convection current

$$\vec{J} = \rho_v \vec{u} \quad ...(4.1.3)$$

ii. The relationship between current elements is given as

$$I\vec{dl} = \vec{K}\,dS = \vec{J}\,dv \quad ...(4.1.4)$$

where, $\vec{K}\,dS$ = Surface current element

and $\vec{J}\,dv$ = Volume current element.

iii. Combining eq. (4.1.3) and (4.1.4), we get

$$I\vec{dl} = \rho_v\,\vec{u}\,dv = dQ\,\vec{u}$$

Alternatively $$I\vec{dl} = \frac{dQ}{dt}\vec{dl} = dQ\frac{\vec{dl}}{dt} = dQ\,\vec{u}$$

$$\therefore \quad I\vec{dl} = dQ\,\vec{u} \quad ...(4.1.5)$$

iv. The force on a current element $I\,\vec{dl}$ in a magnetic field $\vec{B}$ is

$$d\vec{F} = I\vec{dl} \times \vec{B} \quad ...(4.1.6)$$

If the current I is through a closed path L, the force on the circuit is given by

$$\vec{F} = \oint_L I\vec{dl} \times \vec{B} \quad ...(4.1.7)$$

3. **Force between two current elements :**

i. Consider the force between two elements $I_1\vec{dl}_1$ and $I_2\vec{dl}_2$. According to Biot-Savart's law, both current elements produce magnetic fields. Thus the force $d(d\vec{F}_1)$ on element $I_1\vec{dl}_1$ due to field $d\vec{B}_2$ produced by element $I_2\vec{dl}_2$ is as shown in the Fig. 4.1.1.

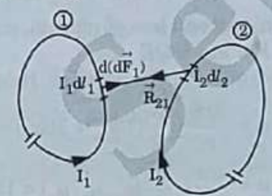

Fig. 4.1.1.

ii. As $$d(d\vec{F}_1) = I_1\vec{dl}_1 \times d\vec{B}_2$$

But from Biot-Savart's law,

$$d\vec{B}_2 = \frac{\mu_0\, I_2\vec{dl}_2 \times \hat{a}_{R_{21}}}{4\pi R_{21}^2}$$

$$\therefore \quad d(d\vec{F}_1) = \frac{\mu_0 I_1\vec{dl}_1 \times (I_2\vec{dl}_2 \times \hat{a}_{R_{21}})}{4\pi R_{21}^2}$$

iii. Total force $\vec{F}_1$ on current loop 1 due to current loop 2 is given as

$$\vec{F}_1 = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1}\oint_{L_2} \frac{\vec{dl}_1 \times (\vec{dl}_2 \times \hat{a}_{R_{21}})}{R_{21}^2}$$

Que 4.2. **A charged particle moves with a uniform velocity 4 m/s in x direction in a region where $E = 20\, a_y$ V/m and $B = B_0\, a_z$ Wb/m^2. Determine B_0 such that the velocity of the particle remains constant.**

AKTU 2018-19, Marks 3.5

Answer

1. If the particle moves with a constant velocity, it implies that its acceleration is zero. In other words, the particle experiences no net force.
2. Hence, $\vec{F} = m\vec{a} = Q(\vec{E} + \vec{u} \times \vec{B}) = 0$

$$Q(20a_y + 4a_x \times B_0 a_z) = 0 \qquad (\because\ a_x \times a_z = -a_y)$$
$$-20a_y = -4B_0 a_y$$
$$\therefore\ B_0 = 5.$$

Que 4.3. **A charged particle of mass 2 kg and charge 3 C starts at point (1, – 2, 0) with velocity $4\, a_x + 3\, a_z$ m/s in an electric field $12\, a_x + 10\, a_y$ V/m. At time $t = 1$ sec, determine the acceleration of the particle, its velocity, kinetic energy of the particles and its position.**

AKTU 2021-22, Marks 10

Answer

1. According to Newton's second law of motion,

$$F = ma = QE$$

where, a is the acceleration of the particle. Hence,

$$a = \frac{QE}{m} = \frac{3}{2}(12a_x + 10a_y) = 18a_x + 15a_y \text{ m/s}^2$$
$$a = \frac{du}{dt} = \frac{d}{dt}(u_x, u_y, u_z) = 18a_x + 15a_y$$

2. Equating components and then integrating, we obtain

$$\frac{du_x}{dt} = 18 \rightarrow u_x = 18t + A \qquad ...(4.3.1)$$
$$\frac{du_y}{dt} = 15 \rightarrow u_y = 15t + B \qquad ...(4.3.2)$$
$$\frac{du_z}{dt} = 0 \rightarrow u_z = C \qquad ...(4.3.3)$$

where, A, B and C are integration constants. But at $t = 0$, $u = 4a_x + 3a_z$. Hence,

$u_x(t = 0) = 4 \rightarrow 4 = 0 + A$ or $A = 4$
$u_y(t = 0) = 0 \rightarrow 0 = 0 + B$ or $B = 0$
$u_z(t = 0) = 3 \rightarrow 3 = C$

3. Substituting the values of A, B and C into eq. (4.3.1) to (4.3.3) give $u(t) = (u_x, u_y, u_z) = (18t + 4,\ 15t,\ 3)$
4. Hence, $u(t = 1 \text{ sec}) = 22a_x + 15a_y + 3a_y + 3a_z$ m/s
5. Kinetic energy (KE) $= \frac{1}{2} m |u|^2 = \frac{1}{2}(2)(22^2 + 15^2 + 3^2)$

$$= 718 \text{ J}$$

6. $$u = \frac{dI}{dt} = \frac{d}{dt}(x, y, z) = (18t + 4,\ 15t,\ 3)$$
7. Equating components yields

$$\frac{dx}{dt} = u_x = 18t + 4 \rightarrow x = 9t^2 + 4t + A_1 \qquad ...(4.3.4)$$
$$\frac{dy}{dt} = u_y = 15t \rightarrow y = 7.5t^2 + B_1 \qquad ...(4.3.5)$$
$$\frac{dz}{dt} = u_z = 3 \rightarrow z = 3t = 3t + C_1 \qquad ...(4.3.6)$$

8. At $t = 0, (x, y, z) = (1, -2, 0)$; hence,

$x(t = 0) = 1 \rightarrow 1 = 0 + A_1$ or $A_1 = 1$
$y(t = 0) = -2 \rightarrow -2 = 0 + B_1$ or $B_1 = -2$
$z(t = 0) = 0 \rightarrow 0 = 0 + C_1$ or $C_1 = 0$

Substituting the values of A_1, B_1 and C_1 into eq. (4.3.4) to eq. (4.3.6), we obtain

$$(x, y, z) = (9t^2 + 4t + 1,\ 7.5t^2 - 2,\ 3t) \qquad ...(4.3.7)$$

9. Hence, at $t = 1$, $(x, y, z) = (14, 5.5, 3)$.
10. By eliminating t in eq. (4.3.7), the motion of the particle may be described in terms of x, y and z.

PART-2

Magnetic Torque and Moment, a Magnetic Dipole.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.4. **What do you understand by magnetic torque and moment ? Derive their equations.**

Answer

A. Magnetic torque :

1. The torque $\vec{T}$ on the loop is the vector product of the force $\vec{F}$ and the moment arm $\vec{r}$.

   i.e., $$\vec{T} = \vec{r} \times \vec{F}$$

2. Let us consider a rectangular loop lies in xy-plane as shown in Fig. 4.4.1. The dimensions of this loop are sides $AB = DC = dx$ and sides $AD = BC = dy$.

3. The value of magnetic field at the centre 'O' of loop is given as $\vec{B}_0$. Since, the loop of differential size, so the value of magnetic fields at all points can be assumed as $\vec{B}_0$. Therefore, there will be no force on the loop.

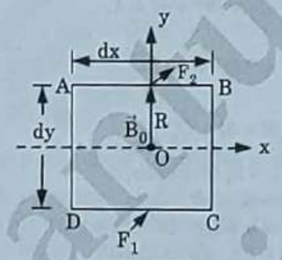

Fig. 4.4.1.

4. Two forces are applied on side DC and AB. The vector force on side DC can be calculated as

$$\partial\vec{F}_1 = Idx \times \hat{a}_x \times \vec{B}_0$$

or,
$$\partial\vec{F}_1 = Idx \times \hat{a}_x \times [B_{0x}\hat{a}_x + B_{0y}\hat{a}_y + B_{0z}\hat{a}_z]$$
$$= Idx[B_{0x}\hat{a}_x \times \hat{a}_x + B_{0y}\hat{a}_x \times \hat{a}_y + B_{0z}\hat{a}_x \times \hat{a}_z]$$
$$= Idx[0 + B_{0y}\hat{a}_z - B_{0z}\hat{a}_y]$$

$$\partial\vec{F}_1 = Idx\ [B_{0y}\hat{a}_z - B_{0z}\hat{a}_y]$$

5. Since this force is applied at the mid-point of side DC. Thus, from Fig. 4.4.1, it is clear that

$$\vec{R} = -\frac{1}{2}dy\,\hat{a}_y$$

6. Thus, torque on the side DC is given as

$$d\vec{T}_{DC} = \vec{R} \times \partial\vec{F}_1$$
$$d\vec{T}_{DC} = -\frac{1}{2}dy\,\hat{a}_y \times Idx\,[B_{0y}\hat{a}_z - B_{0z}\hat{a}_y]$$
$$= -\frac{1}{2}dx\,dy\,I\,[B_{0y}\hat{a}_y \times \hat{a}_z - B_{0z}\hat{a}_y \times \hat{a}_y]$$
$$= -\frac{1}{2}dx\,dy\,I\,[B_{0y}\hat{a}_x - 0]$$
$$d\vec{T}_{DC} = -\frac{1}{2}I\,B_{0y}dx\,dy \cdot \hat{a}_x$$

7. The force on side AB is opposite in direction to force on side DC, therefore the torque on side AB will be opposite in direction but will be of same magnitude.

$$d\vec{T}_{AB} = +\frac{1}{2}I\,B_{0y}dx\,dy \cdot \hat{a}_x$$

8. If the forces F_3 and F_4 on side AD and BC are applied then torques on their sides can also be calculated in similar way.

$$d\vec{T}_{BC} = \frac{1}{2}I\,dx\,dy\,B_{0z}\,\hat{a}_y$$

and
$$d\vec{T}_{AD} = -\frac{1}{2}I\,dx\,dy\,B_{0z}\,\hat{a}_y$$

9. Therefore, the total torque can be written as

$$d\vec{T} = d\vec{T}_{AB} + d\vec{T}_{BC} + d\vec{T}_{DC} + d\vec{T}_{AD}$$
$$d\vec{T} = I\,dx\,dy\,[B_{0x}\hat{a}_y - B_{0y}\hat{a}_x]$$

10. The quantity within the bracket can be represented by a cross product in the form as given below

$$d\vec{T} = I\,dx\,dy\,(\hat{a}_z \times \vec{B}_0)$$

or,
$$d\vec{T} = I\,d\vec{S} \times \vec{B}$$

B. Magnetic dipole moment :

1. It is the product of current and area of the loop. The direction of magnetic dipole moment is perpendicular to the loop. It is given as

$$\vec{m} = I\,S\,\hat{a}_n \quad ...(4.4.1)$$

where,
I = Current
S = Area of loop
$\hat{a}_n$ = Unit normal vector to the plane of the loop.

2. The eq. (4.4.1) can be written for small area as

$$d\vec{m} = I\,d\vec{S}$$

3. In terms of magnetic dipole moment, the torque is given as

$$d\vec{T} = d\vec{m} \times \vec{B}$$

As we assumed earlier that for small loop the magnetic field will be constant throughout the loop.

4. Therefore, for uniform magnetic field, the torque will be given as

$$d\vec{T} = I\,d\vec{S} \times \vec{B}_0$$

or simply, $\vec{T} = I\vec{S} \times \vec{B}_0 = \vec{m} \times \vec{B}_0$

Que 4.5. **Write a short note on magnetic dipole.**

OR

Prove that $\vec{B} = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta)$ for magnetic dipole.

Answer

1. Any magnetic dipole has two distinct poles a north pole and a south pole. Hence, a simple bar magnet is called as magnetic dipole. If this magnet is broken into small pieces then every piece will be a dipole.
2. Similarly, a small current carrying wire or loop is called as dipole, because it has the magnetic poles like bar magnet.

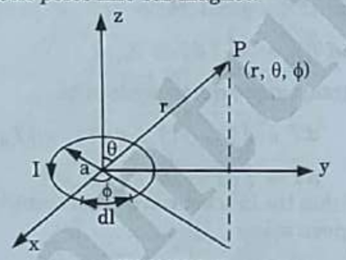

Fig. 4.5.1.

3. Due to current carrying loop, the magnetic vector potential at point 'P' is

$$\vec{A} = \frac{\mu_0 I}{4\pi}\oint\frac{\vec{dl}}{r}$$

$$\vec{A} = \frac{\mu_0 I\pi a^2 \sin\theta\,\hat{a}_\phi}{4\pi r^2} = \frac{\mu_0\,\vec{m} \times \hat{a}_r}{4\pi r^2}$$

where, $\vec{m}$ = Magnetic dipole moment $= I\pi a^2\hat{a}_z$

4. We know that

$$\hat{a}_z \times \hat{a}_r = \sin\theta \cdot \hat{a}_\phi$$

5. The flux density $\vec{B}$ can also be determined as

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{B} = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta)$$

PART-3

Magnetization in Materials.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.6. **What is magnetic susceptibility ? What is magnetization ?**

Answer

A. Magnetic susceptibility :

1. In the magnetic materials, the magnetic dipoles are present corresponding to the atoms. These magnetic dipoles are assumed to be oriented randomly at rest (initially).
2. The electron by spinning about its own axis creates a magnetic field and magnetic dipole moments are also created by the electron orbiting about the positive nucleus.
3. When the external magnetic field is applied to the material, the magnetic dipoles inside the material are re-oriented according to the direction of applied external magnetic field.
4. The application of external magnetic field affects the velocities of the orbiting electrons. Thus, a small magnetic moment for the atom is created that according to Lenz's law will oppose the applied magnetic field.
5. The measurement of this re-orientation is called magnetic susceptibility. This is denoted by the symbol χ_m.

Fig. 4.6.1. Randomly oriented magnetic dipoles in a materials.

6. The materials which come under this category are known as paramagnetic materials. These materials are iron, nickel, cobalt, etc.

B. Magnetization (*M*) :

1. It is simply given as total magnetization, the values of this is given as the magnetic dipole moment per unit volume of a material in a volume Δv.

2. If there are n atoms in a given volume. The total magnetization is defined as Δv and j^{th} atom has a magnetic moment (m_j).

$$\vec{M} = \lim_{\Delta v \to 0} \frac{1}{\Delta v} \sum_{j=1}^{j=n} \vec{m}_j \text{ A/m}^2 \quad ...(4.6.1)$$

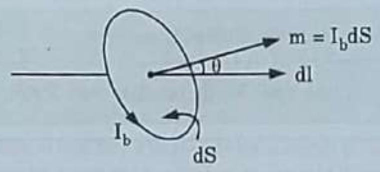

Fig. 4.6.2. An alignment of magnetic dipole moment due to external magnetic field.

3. From Fig. 4.6.2

$$dI_b = nI_b d\vec{S} \cdot \vec{dl} = \vec{M} \cdot \vec{dl} \quad ...(4.6.2)$$

then
$$I_b = \oint \vec{M} \cdot \vec{dl} \quad ...(4.6.3)$$

4. By Ampere's circuital law, the total current I_T,

$$I_T = I_b + I = \oint \vec{H} \cdot d\vec{l} \quad ...(4.6.4)$$

$$I_T = \oint \frac{\vec{B}}{\mu_0} \cdot d\vec{l} \quad ...(4.6.5)$$

5. From eq. (4.6.3), (4.6.4), and (4.6.5)

$$I = I_T - I_b$$

$$I = \oint \left(\frac{\vec{B}}{\mu_0}\right) \cdot d\vec{l} - \oint \vec{M} \cdot d\vec{l}$$

$$I = \oint \left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) \cdot d\vec{l} \quad ...(4.6.6)$$

6. Compare eq. (4.6.6) with the expression of Ampere's circuital law

$$I = \oint \vec{H} \cdot d\vec{l}$$

Thus,
$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

7. For linear, isotropic magnetic materials

$$\vec{M} = \chi_m \vec{H}$$

The quantity χ_m is dimensionless and is called magnetic susceptibility of the medium.

PART-4

Magnetic Boundary Conditions.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.7. **What is magnetic dipole ? Find magnetic vector potential. Explain the complete magnetic boundary conditions. Derive all tangential and normal components.** **AKTU 2021-22, Marks 10**

OR

Explain magnetic boundary conditions.

Answer

A. **Magnetic dipole :** Refer Q. 4.5, Page 4–8B, Unit-4.

B. **Magnetic vector potential :** Refer Q. 3.12, Page 3–17B, Unit-3.

C. **Boundary conditions on magnetic fields :** In a single medium, the magnetic field is continuous. Here we will find the boundary conditions for magnetic fields at an interface of two media.

i. **Condition for normal component of magnetic field :**

1. Let us consider two different media with permeabilities μ_1 and μ_2. Both media are linear, isotropic and homogenous.
2. In Fig. 4.7.1, we have two different media with μ_1 and μ_2 respectively. We choose an elementary cylindrical Gaussian surface at the interface of two media so that half of cylinder is in media-1 and other half in media-2.

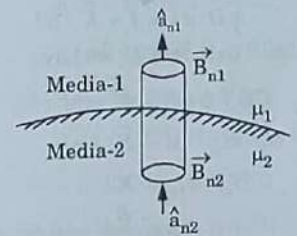

Fig. 4.7.1.

3. Now apply Gauss's law for magnetic field.

$$\nabla \cdot \vec{B} = 0$$

$$\oint_s \vec{B} \cdot d\vec{S} = 0$$

$$[\vec{B}_{n1}\, \hat{a}_{n1}\, \Delta S - \vec{B}_{n2}\, \hat{a}_{n2}\, \Delta S] = 0$$

$$(\vec{B}_{n1} - \vec{B}_{n2}) \cdot \Delta S = 0$$

4. Since, ΔS represents small area enclosing volume which cannot be equal to zero, *i.e.*, $\Delta S \neq 0$

$$\vec{B}_{n1} - \vec{B}_{n2} = 0$$

or $$\mu_1 \vec{H}_{n1} - \mu_2 \vec{H}_{n2} = 0 \quad ...(4.7.1)$$

i.e., normal component of magnetic flux density ($\vec{B}$) is continuous across the boundary interface between two media.

5. From eq. (4.7.1)

$$\frac{\vec{H}_{n1}}{\vec{H}_{n2}} = \frac{\mu_2}{\mu_1}$$

i.e., normal component of magnetic field intensity is discontinuous.

ii. Condition for tangential component of magnetic field :

1. At the boundary, there exists a sheet current of density $\vec{K}$ shown in Fig. 4.7.2.

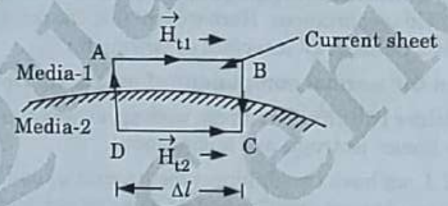

Fig. 4.7.2.

2. Apply Ampere's circuital law,

$$\oint \vec{H} \cdot d\vec{l} = I = \vec{K} \cdot \Delta l \quad ...(4.7.2)$$

3. Considering the closed path $ABCD$, we have

$$H_{t1} \cdot AB - H_{t2} \cdot CD + 0 + 0 = \vec{K} \cdot \Delta l$$

$$H_{t1} \cdot \Delta l - H_{t2} \cdot \Delta l = \vec{K} \cdot \Delta l$$

$$H_{t1} - H_{t2} = \vec{K}$$

$$H_{t1} = \frac{B_{t1}}{\mu_1} \text{ and } H_{t2} = \frac{B_{t2}}{\mu_2}$$

$$\frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = \vec{K} \quad ...(4.7.3)$$

4. Tangential component of magnetic field intensity is discontinuous across the boundary and the value of discontinuity is equal to the surface current density ($\vec{k}$).

5. If current density $\vec{k}$ is zero,

$$\vec{H}_{n1} - \vec{H}_{n2} = 0 \quad ...(4.7.4)$$

Thus, the tangential component of $\vec{H}$ is continuous across the boundary between two media provided that the boundary has no current sheet.

6. From eq. (4.7.4), $\vec{H}_{n1} = \vec{H}_{n2}$

$$\frac{\vec{B}_{t1}}{\mu_1} = \frac{\vec{B}_{t2}}{\mu_2}$$

Thus, tangential component of $\vec{B}$ is discontinuous since μ_1 is not equal to μ_2.

Que 4.8. **Given that $\vec{H}_1 = -2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z$ A/m in region $y - x - 2 < 0$, where $\mu_1 = 5\mu_0$. Calculate $\vec{M}_1$ and $\vec{B}_1$.**

Answer

1. If we let the surface of plane be described by $f(x, y) = y - x - 2$, a unit vector normal to plane is given by

$$\hat{a}_n = \frac{\vec{\nabla} f}{|\nabla f|} = \frac{\hat{a}_y - \hat{a}_x}{\sqrt{2}}$$

2. $$\vec{M}_1 = \chi_{m1} \vec{H}_1 = (\mu_{r1} - 1)\vec{H}_1$$

$$= (5 - 1)(-2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z)$$

$$= (-8\hat{a}_x + 24\hat{a}_y + 16\hat{a}_z) \text{ A/m}$$

3. $$\vec{B}_1 = \mu_1 \vec{H}_1 = \mu_0 \mu_{r1} \vec{H}_1$$

$$= 4\pi \times 10^{-7}(5)(-2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z)$$

$$= (-12.57\hat{a}_x + 37.7\hat{a}_y + 25.13\hat{a}_z)\ \mu\text{Wb/m}^2$$

Que 4.9. **Write down the boundary condition for current density.**

Answer

1. When current obliquely crosses an interface between two media with different conductivities, the current density vector changes both in direction and magnitude.
2. The equations for steady current density $\vec{J}$ are :

Differential form	Integral form	
$\nabla \cdot \vec{J} = 0$	$\oint \vec{J} \cdot dS = 0$	...(4.9.1)
$\nabla \times \left(\dfrac{\vec{J}}{\sigma}\right) = 0$	$\oint_C \dfrac{1}{\sigma} \vec{J} \cdot \vec{dl} = 0$	...(4.9.2)

3. The divergence equation is same and the curl equation is obtained by combining Ohm's law ($\vec{J} = \sigma \vec{E}$) with $\nabla \times \vec{E} = 0$.
4. By applying eq. (4.9.1) and (4.9.2) at the interface between two ohmic media with conductivities σ_1 and σ_2, we obtain boundary condition for normal and tangential component $\vec{J}$.
5. The normal component of a divergenceless vector field is continuous.

   Hence from $\nabla \cdot \vec{J} = 0$, we have,
   $$\vec{J}_{1n} = \vec{J}_{2n} \text{ A/m}^2$$
6. Similarly tangential component of a curl free vector field is continuous across an interface. We conclude $\nabla \times (\vec{J}/\sigma) = 0$, *i.e.*,
   $$\frac{\vec{J}_{1t}}{\vec{J}_{2t}} = \frac{\sigma_1}{\sigma_2}$$
7. It states that the ratio of tangential component of $\vec{J}$ at two sides of an interface is equal to the ratio of conductivities.

PART-5

Inductors and Inductances, Magnetic Energy.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.10. **Write short notes on following :**

i. **Inductor**
ii. **Magnetic materials.**

Answer

i. **Inductor :**

1. An inductor is a passive component that stores energy in the form of magnetic field and is also called as inductance.

   Example : Coil and solenoid.
   $$\text{Inductance } (L) = \frac{\text{Total magnetic flux linkage}}{\text{Current passing through it}} = \frac{\phi_T}{I} = \frac{N\phi}{I}$$
2. There are two types of inductances :

A. **Self inductance :** Self inductance of a circuit is the property of the circuit by which any change in the circuit current induces an e.m.f. in the circuit to oppose the change of current.

We know that
$$\phi \propto I \text{ (current)}$$
$$\phi = LI$$
$$L = \frac{\phi}{I} = \frac{\text{Total flux}}{\text{Current}}$$

B. **Mutual inductance :**

a. Mutual inductance occurs in at least two circuits. Flow of current in a circuit due to change in current in neighbouring circuit is due to mutual inductance.

b. Let I_1 = Current in circuit 1.

I_2 = Current in circuit 2.

ϕ_1 = Magnetic flux linkage in circuit 1 by the current (I_2) flowing in circuit 2.

ϕ_2 = Magnetic flux linkage in circuit 2 by the current (I_1) flowing in circuit 1.

M_{12} = Mutual inductance between circuit 1 and 2.

M_{21} = Mutual inductance between circuit 2 and 1.
$$M_{12} = \frac{\phi_1}{I_2} \text{ and } M_{21} = \frac{\phi_2}{I_1}$$

3. Mutual inductance between circuit 2 and 1 is defined as flux linked with a circuit when a unit current flows in neighbouring circuit.

ii. **Magnetic materials :** There are three types of magnetic materials :

1. **Diamagnetic :** When a diamagnetic material is placed in a magnetic field, it becomes weakly magnetized in a direction opposite to the direction of external magnetic field. This property of material is known as diamagnetism.
   Examples : Bismuth, silver, copper and hydrogen.
2. **Paramagnetic :**
   a. In paramagnetic materials the magnetic dipoles are already present. These materials are permanently magnetized but dipoles are randomly oriented and have a low net magnetism.
   b. When these materials are placed in external magnetic field, the dipoles orient themselves in the direction of external magnetic field. This property is known as paramagnetism.
   Examples : Aluminium, platinum and oxygen.
3. **Ferromagnetic :** When a ferromagnetic material is placed in external magnetic field, it strongly attracts the magnetic lines of force and the domains orient themselves in the direction of the field to increase the flux produced by the external field. This property is known as ferromagnetism.
   Examples : Iron, cobalt, nickel.

Que 4.11. Derive an expression for inductance of solenoid and toroid.

Answer

A. Expression for inductance of solenoid :

1. A solenoid is the cylindrical structure wrapped by wire. It is of infinite length.

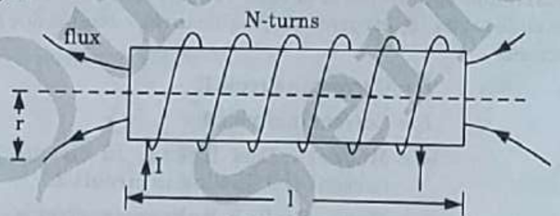

Fig. 4.11.1.

2. Suppose dl = Differential length of coil.
   N = Number of turns in a solenoid.
   r = Radius of coil.
   I = Current through the coil.
3. Using Ampere's circuit law, we have

$$\oint_{circle} \vec{H}\cdot d\vec{l} = I$$

But $$\oint_{circle} \vec{H}\cdot d\vec{l} = \int H\, dl \cos\theta = \int H\, dl$$

[$\because$ $\vec{H}$ and $d\vec{l}$ are parallel to each other]

$$\oint_{circle} \vec{H}\cdot d\vec{l} = H\int dl = I \quad ...(4.11.1)$$

4. For N number of turns in the solenoid, the current enclosed by the circle is

$$I_{total} = NI$$

Therefore, from eq. (4.11.1) we get

$$\oint_{circle} \vec{H}\cdot d\vec{l} = H(2\pi r) = NI$$

or, $$H = \frac{NI}{2\pi r} \quad ...(4.11.2)$$

5. For a constant current carrying conductor

$$H \propto \frac{1}{r}$$

It means that the magnetic field due to a solenoid is zero at all points except within the core.

6. Let $l = 2\pi r$, then eq. (4.11.2) will become

$$H = \frac{NI}{l}$$

or, $$|B| = \mu H = \frac{\mu NI}{l} \quad ...(4.11.3)$$

7. The total flux linkage by a long solenoid, $\phi = N\psi_m = NBA$
   where, A is area of solenoid.

$$\phi = \frac{\mu N^2 IA}{l} \text{ Henry}$$

8. Self inductance of coil is $L = \frac{\phi}{I}$

$$L = \frac{\mu N^2 A}{l} \text{ Henry}$$

B. Expression for inductance of toroid :

1 When a long solenoid is bent into a circle and closed on itself, a toroid is obtained. Let I be the current flowing in the toroid and $\vec{B}$ is the magnetic field produced at every point of circular path of radius R.

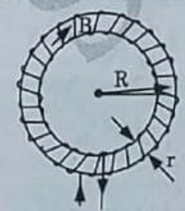

Fig. 4.11.2.

2. According to Ampere's circuit law, flux produced at a distance R is

$$B = \frac{\mu I}{2\pi R} \text{ Wb/m}^2$$

3. As toroid has N turns, then

$$B = \frac{\mu NI}{2\pi R}$$

4. Flux linkage is $\phi = N\psi_m = NBA$

$$\phi = N\frac{\mu NI}{2\pi R}A = \frac{\mu N^2 IA}{2\pi R}$$

So, $$\phi = \frac{\mu N^2 I \pi r^2}{2\pi R} = \frac{\mu N^2 I r^2}{2R}$$

5. Self inductance of toroid is $L = \frac{\phi}{I} = \frac{\mu N^2 r^2}{2R}$

Que 4.12. **Explain the concept of magnetic energy.**

OR

Prove the magnetostatic energy is given by

$$W_m = \frac{1}{2}\int_v \mu H^2 dv$$

AKTU 2017-18, Marks 07

Answer

A. Energy stored in magnetic field :

1. An inductor stores energy when a current through an inductance coil is gradually changed from zero to the maximum value I.
2. The energy change of inductor is opposed by the self induced e.m.f., produced due to this change. This energy is stored in magnetic field of the coil and is later on recovered when that field collapse.

$$\text{Energy stored} = \text{Work done} = W = \int_0^t P\,dt \text{ joules.}$$

where, P is the power and is equal to VI.

So, $$W = \int_0^t VI\,dt \quad ...(4.12.1)$$

3. From circuit theory, voltage across an inductor is given by

$$V = L\frac{dI}{dt} \quad ...(4.12.2)$$

From eq. (4.12.1) and (4.12.2), we have

$$W = \int_0^t LI\frac{dI}{dt}dt$$

$$W = \int_0^t LI\,dI = L\left[\frac{I^2}{2}\right]_0^I$$

4. Energy stored in magnetic field, $W = \frac{1}{2}LI^2$

B. Energy density in magnetic field :

1. Consider a small unit cube of side Δl and volume $\Delta v = (\Delta l)^3$ as shown in Fig. 4.12.1. Each cube can be considered as magnetic field cell transmission line of length Δl.

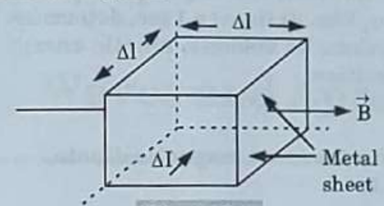

Fig. 4.12.1.

2. Each cell has an inductance $\Delta L = \mu\Delta l$ ...(4.12.3)
3. The field H is related to the current ΔI by

$$H = \frac{\Delta I}{\Delta l} \quad ...(4.12.4)$$

4. We know that the energy stored in each cell is given by

$$\Delta W_m = \frac{1}{2}\Delta L\,(\Delta I)^2 \text{ joules}$$

5. From eq. (4.12.3) and (4.12.4), we have

$$\Delta W_m = \frac{1}{2}\mu(\Delta l)(H\cdot\Delta l)^2 = \frac{1}{2}\mu H^2(\Delta l)^3 = \frac{1}{2}\mu H^2(\Delta v)$$

6. The magnetostatic energy density w_m (in J/m³) is defined as

$$w_m = \lim_{\Delta v\to 0}\frac{\Delta W_m}{\Delta v} = \frac{1}{2}\mu H^2$$

Hence, $$w_m = \frac{1}{2}\mu H^2 = \frac{1}{2}B\cdot H = \frac{B^2}{2\mu}$$

7. Thus the energy in a magnetostatic field in a linear medium is

$$W_m = \int w_m\,dv$$

$$W_m = \frac{1}{2}\int \vec{B}\cdot\vec{H}\,dv = \frac{1}{2}\int \mu H^2\,dv$$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Discuss the ways in which force due to magnetic fields can be experienced.

Ans. Refer Q. 4.1.

Q. 2. A charged particle of mass 2 kg and charge 3 C starts at point (1, – 2, 0) with velocity $4\,a_x + 3\,a_z$ m/s in an electric field $12\,a_x + 10\,a_y$ V/m. At time $t = 1$ sec, determine the acceleration of the particle, its velocity, kinetic energy of the particles and its position.

Ans. Refer Q. 4.3.

Q. 3. Write a short note on magnetic dipole.

Ans. Refer Q. 4.5.

Q. 4. What is magnetic dipole ? Find magnetic vector potential. Explain the complete magnetic boundary conditions. Derive all tangential and normal components.

Ans. Refer Q. 4.7.

Q. 5. Write short notes on following :

i. Inductor

ii. Magnetic materials.

Ans. Refer Q. 4.10.

Q. 6. Prove the magnetostatic energy is given by

$$W_m = \frac{1}{2}\int_v \mu H^2 dv$$

Ans. Refer Q. 4.12.

☺☺☺

5 UNIT Waves and Applications

CONTENTS

PART-1

Maxwell's Equation, Faraday's Law, Transformer and Motional Electromotive Forces.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.1. **Derive and explain differential form of Faraday's law of electromagnetic induction in vector form.**

AKTU 2019-20, Marks 10

OR

Derive Faraday's law of induction. Explain the concept of transformer and motional electromotive force.

OR

Write a short note on
Magnetic scalar and vector potential, Faraday law of electromagnetic induction. **AKTU 2018-19, Marks 3.5**

Answer

A. Faraday's law :

1. Faraday discovered that the induced emf, V_{emf} in any closed circuit is equal to the time rate of change of magnetic flux linkage by the circuit.
2. Let ψ is the flux due to electromagnetic field. If there is the variation in fields then there will be changes in the magnetic flux and an emf will be induced in a coil and it is given as

$$V_{emf} = -\frac{d\psi}{dt} \text{ volts} \quad ...(5.1.1)$$

3. If there are N turns in a coil then the induced emf will be,

$$V_{emf} = -N\frac{d\psi}{dt} \text{ volts} \quad ...(5.1.2)$$

4. Negative sign signifies that the induced emf always opposes the cause due to which it is produced.
5. Suppose $\vec{E}$ is applied electric field and $d\vec{l}$ is small length of coil. Then induced emf will be

$$V_{emf} = \oint \vec{E} \cdot d\vec{l} \quad ...(5.1.3)$$

6. The magnetic flux ψ, due to magnetic field $\vec{B}$, passing through a specified area $d\vec{S}$ is given as

$$\psi = \oint_S \vec{B} \cdot d\vec{S} \quad ...(5.1.4)$$

7. Using eq. (5.1.3) and (5.1.4), the eq. (5.1.1) can be written as

$$V_{emf} = \oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\left\{\oint_S \vec{B} \cdot d\vec{S}\right\} \quad ...(5.1.5)$$

B. Transformer emf (stationary loop in a time-varying $\vec{B}$ field) :

1. The emf induced by the time varying current (producing the time-varying $\vec{B}$ field) in a stationary loop is referred to as transformer emf, since it is due to transformer action.
2. Let us consider a stationary conducting loop in a time-varying magnetic field $\vec{B}$ (Fig. 5.1.1) then eq. (5.1.5) becomes,

$$V_{emf} = \oint_L \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \quad ...(5.1.6)$$

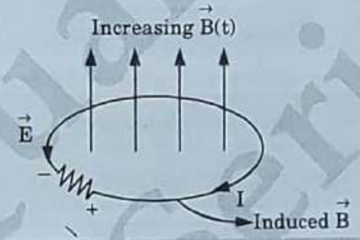

Fig. 5.1.1. Induced emf due to a stationary loop in a time-varying $\vec{B}$ field.

3. By applying Stoke's theorem to the middle term in eq. (5.1.6), we get

$$\int_S (\nabla \times \vec{E}) \cdot d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \quad ...(5.1.7)$$

For the two integrals to be equal, their integrands must be equal,

i.e., $$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad ...(5.1.8)$$

C. Motional emf (moving loop in a static $\vec{B}$ field) :

1. When a conducting loop is moving in a static $\vec{B}$ field, an emf is induced

in the loop. The force on a charge moving with uniform velocity $\vec{u}$ in a magnetic field $\vec{B}$ is given by

$$\vec{F}_m = Q\vec{u} \times \vec{B} \quad ...(5.1.9)$$

2. The motional electric field $\vec{E}_m$ is given as

$$\vec{E}_m = \frac{\vec{F}_m}{Q} = \vec{u} \times \vec{B} \quad ...(5.1.10)$$

3. If a conducting loop is considered moving with uniform velocity $\vec{u}$, consisting of large number of free electrons, the emf induced in the loop is

$$V_{emf} = \oint_L \vec{E}_m \cdot \vec{dl} = \oint_L (\vec{u} \times \vec{B}) \cdot \vec{dl} \quad ...(5.1.11)$$

4. This type of emf is called motional emf because it is due to motional action. It is a kind of emf found in electric machines such as motors, generators and alternators.

D. **Magnetic scalar and vector potential :** Refer 3.12, Page 3–17B, Unit-3.

PART-2

Displacement Current.

Questions-Answers
Long Answer Type and Medium Answer Type Questions

Que 5.2. **Explain the concept of displacement current in an electrical circuit. Also determine the condition when conduction current becomes equal to displacement current.**

AKTU 2019-20, Marks 10

Answer

A. **Displacement current :**

1. In order to understand the concept of displacement current, let's consider Maxwell's curl equation for magnetic fields for time-varying conditions.
2. Thus, for static EM fields,

$$\nabla \times \vec{H} = \vec{J} \quad ...(5.2.1)$$

Taking the divergence of each side of eq. (5.2.1)

$$\nabla \cdot (\nabla \times \vec{H}) = 0 = \nabla \cdot \vec{J} \quad ...(5.2.2)$$

3. As we know, the divergence of the curl of a vector is zero, so $\nabla \cdot \vec{J}$ is also zero. However, the equation of continuity is

$$\nabla \cdot \vec{J} = \frac{-\partial \rho_v}{\partial t} \neq 0 \quad ...(5.2.3)$$

4. As eq. (5.2.2) and (5.2.3) are incompatible for time-varying condition, thus we modify eq. (5.2.1) to agree with eq. (5.2.3) by adding a term to eq. (5.2.1)

i.e.,
$$\nabla \times \vec{H} = \vec{J} + \vec{J}_d \quad ...(5.2.4)$$

where, $\vec{J}_d$ is to be determined and defined.

5. Again taking the divergence of eq. (5.2.4), we get

$$\nabla \cdot (\nabla \times \vec{H}) = 0 = \nabla \cdot \vec{J} + \nabla \cdot \vec{J}_d \quad ...(5.2.5)$$

$$\therefore \quad \nabla \cdot \vec{J}_d = -\nabla \cdot \vec{J} = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t}(\nabla \cdot \vec{D})$$

$$\nabla \cdot \vec{J}_d = \nabla \cdot \frac{\partial \vec{D}}{\partial t}$$

$$\therefore \quad \vec{J}_d = \frac{\partial \vec{D}}{\partial t} \quad ...(5.2.6)$$

6. Substituting eq. (5.2.6) into eq. (5.2.4), we get

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad ...(5.2.7)$$

Eq. (5.2.7) is Maxwell's equation (based on Ampere's circuit law) for a time-varying field.

7. The term $\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$ is known as displacement current density and $\vec{J}$ is conduction current density.
8. Based on displacement current density, the displacement current is defined as,

$$I_d = \int \vec{J}_d \cdot d\vec{S} = \int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S}$$

B. **Condition :** The displacement current in the space between the plates of capacitor during charging is equal to the conduction current.

Que 5.3. **A parallel plate capacitor with a plate area of 5 cm^2 and plate separation of 3 mm has a voltage 50 sin $10^3 t$ V applied to its plates. Calculate the displacement current assuming $\epsilon = 4\epsilon_0$.**

Answer

Given : $d = 3$ mm, $S = 5$ cm^2, $\epsilon = 4\epsilon_0$ and $V = 50 \sin 10^3 t$ V
To Find : I_d

1. We know that,

$$D = \epsilon E = \epsilon \frac{V}{d}$$

2. The displacement current density is given by

$$J_d = \frac{\partial D}{\partial t} = \frac{\epsilon}{d}\frac{dV}{dt}$$

3. Hence, displacement current

$$I_d = J_d \cdot S = \frac{\epsilon S}{d}\frac{dV}{dt}$$

$$I_d = 4 \times \frac{10^{-9}}{36\pi} \times \frac{5 \times 10^{-4}}{3 \times 10^{-3}} 10^3 \times 50 \cos 10^3 t \qquad (\because \epsilon = 4\epsilon_0)$$

$$I_d = 2.9473 \times 10^{-7} \cos 10^3 t \text{ A}$$

Que 5.4. **In a material for which $\sigma = 5$ S/m and $\epsilon_r = 1$, the electrical field intensity is $\vec{E} = 250 \sin 10^{10} t$ V/m. Find conduction and displacement current densities and the frequency at which both have equal magnitude.**

Answer

Given : $\vec{E} = 250 \sin 10^{10} t$ V/m, $\epsilon_r = 1$, $\sigma = 5$ /m
To Find : J_c, J_d and f.

1. The conduction current density is,

$$\vec{J}_c = \sigma \vec{E} = 5 \times 250 \times \sin(10^{10} t)$$
$$= 1250 \sin(10^{10} t) \text{ A/m}^2$$

2. The displacement current density is

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t} = \epsilon_0 \epsilon_r \frac{\partial}{\partial t}(\vec{E})$$

$$= 8.85 \times 10^{-12} \times 1 \times 250 \times 10^{10} \cos(10^{10} t) = 22.125 \cos(10^{10} t) \text{ A/m}^2$$

3. The displacement current density in frequency domain is,

$$\vec{J}_d = j\omega\epsilon \vec{E}$$

The two current densities are equal, when $\omega\epsilon = \sigma$

or $$f = \frac{\sigma}{2\pi\epsilon} = \frac{5}{2\pi \times 1 \times 8.85 \times 10^{-12}}$$
$$= 90 \times 10^9 \text{ Hz} = 90 \text{ GHz}$$

PART-3

Maxwell's Equation in Final Form.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.5. **Write and explain all forms of all Maxwell's equation in detail.** **AKTU 2019-20, Marks 10**

OR

State and explain Maxwell's equations for time varying fields in differential and integral form.

Answer

A. **Maxwell's equations for static fields :** Refer Q. 3.11, Page 3–15B, Unit-3.

B. **Maxwell's equations for time-varying fields :**

a. **Maxwell's first equation :**

1. $$\oint_S \vec{D} \cdot \vec{dS} = \int_v \rho_v \, dv \quad \text{(Integral form)} \qquad ...(5.5.1)$$

2. $$\nabla \cdot \vec{D} = \rho_v \quad \text{(Differential form)} \qquad ...(5.5.2)$$

3. The derivation is same as for the static fields.

b. **Maxwell's second equation :**

1. It is derived from Faraday's law. According to this law

$$\oint_L \vec{E} \cdot \vec{dl} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \quad \text{(Integral form)} \qquad ...(5.5.3)$$

2. Applying Stoke's theorem to eq. (5.5.3) gives

$$\oint_L \vec{E} \cdot \vec{dl} = \int_S (\nabla \times \vec{E}) \cdot d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t} d\vec{S} \qquad ...(5.5.4)$$

3. Comparing the two surface integrals in eq. (5.5.4) gives

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{(Differential or point form)} \qquad ...(5.5.5)$$

4. Eq. (5.5.3) is the integral form and eq. (5.5.5) is the differential form of Maxwell's equation.

c. **Maxwell's third equation :**

1. It is derived from Ampere's circuit law. According to this law

$$\oint \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{S} \quad ...(5.5.6)$$

2. On adding displacement current to right side of eq. (5.5.6) we have

$$\oint \vec{H} \cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \cdot d\vec{S} \text{ (Integral form)} \quad ...(5.5.7)$$

3. Applying Stoke's theorem to eq. (5.5.7) gives

$$\oint \vec{H} \cdot d\vec{l} = \int_S (\nabla \times \vec{H}) \cdot d\vec{S} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \cdot d\vec{S} \quad ...(5.5.8)$$

4. Comparing two surface integrals in eq. (5.5.8) gives

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \text{ (Differential or point form)} \quad ...(5.5.9)$$

5. Eq. (5.5.7) is the integral form and eq. (5.5.9) is the differential (or point) form of Maxwell's equation.

d. Maxwell's fourth equation :

1. $\int_S \vec{B} \cdot d\vec{S} = 0$ (Integral form)
2. $\nabla \cdot \vec{B} = 0$ (Differential or point form)
3. The derivation is same as for the static fields.

C. Maxwell's equations for time harmonic fields : Assuming time factor $e^{j\omega t}$, the Maxwell's equation is given by

Point form	Integral form
$\nabla \cdot \vec{D}_s = \rho_{vs}$	$\oint \vec{D}_s \cdot d\vec{S} = \oint \rho_{vs} dv$
$\nabla \cdot \vec{B}_s = 0$	$\oint \vec{B}_s \cdot d\vec{S} = 0$
$\nabla \times \vec{E}_s = -j\omega \vec{B}_s$	$\oint \vec{E}_s \cdot d\vec{l} = -j\omega \int \vec{B}_s \cdot d\vec{S}$
$\nabla \times \vec{H}_s = J_s + j\omega D_s$	$\oint \vec{H}_s \cdot d\vec{l} = \int (\vec{J}_s + j\omega \vec{D}_s) \cdot d\vec{S}$

Que 5.6. Explain all forms of Maxwell's equations in time varying conditions with its physical significance.

AKTU 2018-19, Marks 07

OR

State and explain Maxwell's equations for time varying fields in differential and integral forms and their significance.

AKTU 2017-18, Marks 07

Answer

A. **Maxwell's equation time varying fields :** Refer Q. 5.5, Page 5–7B, Unit-5.

B. **Physical significance :**

1. **Maxwell's first equation :** This Maxwell equation implies that the electric flux lines are not continuous; they originate from the positive charges and terminate on the negative charges.
2. **Maxwell's second equation :** This Maxwell equation inter-relates the magnetic and electric fields and shows that a time-varying magnetic field can produce an electric field.
3. **Maxwell's third equation :** This Maxwell equation signifies that conduction current as well as a changing electric field produces a magnetic field.
4. **Maxwell's fourth equation :** This implies that the number of magnetic flux lines of force entering any region must be equal to the number of lines of force coming out of that region.

PART-4

Electromagnetic Wave Propagation : Wave Propagation in Loss Dielectric.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.7. What do you mean by lossy dielectric and explain the wave propagation in lossy dielectric.

OR

Discuss the solution of plane wave equation in conducting media (lossy dielectric).

OR

Derive an expression for attenuation constant, propagation constant and intrinsic impedance of an EM wave when it is propagating through a lossy dielectric medium.

AKTU 2018-19, Marks 07

Answer

1. It is a medium in which an EM wave, as it propagates, loses power owing to imperfect dielectric.
2. A lossy dielectric is a partially conducting medium (imperfect dielectric or imperfect conductor) with $\sigma \neq 0$.

3. Lets consider a linear, isotropic, homogeneous, lossy dielectric medium that is charge free ($\rho_v = 0$).
4. Considering the time factor $e^{j\omega t}$, Maxwell's equations become

$$\nabla \cdot \vec{E}_s = 0 \quad ...(5.7.1)$$

$$\nabla \cdot \vec{H}_s = 0 \quad ...(5.7.2)$$

$$\nabla \times \vec{E}_s = -j\omega\mu \vec{H}_s \quad ...(5.7.3)$$

$$\nabla \times \vec{H}_s = (\sigma + j\omega\epsilon)\vec{E}_s \quad ...(5.7.4)$$

Taking curl on both sides of eq. (5.7.3), we get

$$\nabla \times (\nabla \times \vec{E}_s) = -j\omega\mu\,(\nabla \times \vec{H}_s) \quad ...(5.7.5)$$

5. As we know $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla . \vec{A}) - \nabla^2 \vec{A}$, thus applying this vector identity to eq. (5.7.5) and invoking eq. (5.7.1) and (5.7.4), we get

$$\nabla(\nabla \cdot \vec{E}_s) - \nabla^2 \vec{E}_s = -j\omega\mu(\sigma + j\omega\epsilon)\vec{E}_s \quad ...(5.7.6)$$

As $\nabla(\nabla \cdot \vec{E}_s) = 0$ and $j\omega\mu(\sigma + j\omega\epsilon) = \gamma^2$ ...(5.7.7)

6. Thus, eq. (5.7.6) becomes

$$\nabla^2 \vec{E}_s - \gamma^2 \vec{E}_s = 0 \quad ...(5.7.8)$$

Here, γ is in reciprocal meters and known as propagation constant of the medium.

Similarly $\nabla^2 \vec{H}_s - \gamma^2 \vec{H}_s = 0$ ...(5.7.9)

Eq. (5.7.8) and (5.7.9) are known as Helmholtz's equations or vector wave equations.

7. Let's assume $\gamma = \alpha + j\beta$ ...(5.7.10)

We obtain attenuation constant (α) and phase constant (β) from eq. (5.7.7) and (5.7.10)

$\therefore$ $-\text{Re}\,\gamma^2 = \beta^2 - \alpha^2 = \omega^2\mu\epsilon$ ...(5.7.11)

and $|\gamma^2| = \beta^2 + \alpha^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\epsilon^2}$ ...(5.7.12)

8. From eq. (5.7.11) and (5.7.12), we get

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\epsilon}\right]^2} - 1\right]} \quad ...(5.7.13)$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\epsilon}\right]^2} + 1\right]} \quad ...(5.7.14)$$

9. Considering only E_{ys} component of E_s varying w.r.t. x

$$\nabla \times \vec{E}_s = \frac{\partial E_{ys}}{\partial x}\hat{z} = \hat{z}\frac{\partial}{\partial x}(E_m e^{-\gamma x}) = -\hat{z}\,\gamma\,E_m e^{-\gamma x}$$

Substituting in eq. (5.7.3), we get

$$-j\omega\mu\vec{H}_s = -\hat{z}\,\gamma E_m e^{-\gamma x}$$

$\vec{H}_s$ has H_{zs} component corresponding to E_{ys}

$$H_{zs} = \frac{\gamma}{j\omega\mu}E_{ys}$$

10. Hence, the intrinsic impedance

$$\eta = \frac{E_{ys}}{H_{zs}} = \frac{j\omega\mu}{\gamma} \quad ...(5.7.15)$$

Putting γ in eq. (5.7.15), we get

$$\eta = \frac{j\omega\mu}{j\omega\sqrt{\mu\epsilon}\sqrt{1 - \frac{j\sigma}{\omega\epsilon}}} = \sqrt{\frac{\mu}{\epsilon\left(1 + \frac{\sigma}{j\omega\epsilon}\right)}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = |\eta|\angle\theta_\eta = |\eta|e^{j\theta_\eta}$$

$$\eta = \frac{\sqrt{\frac{\mu}{\epsilon}}}{\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}};\ \tan 2\theta_\eta = \frac{\sigma}{\omega\epsilon}$$

Que 5.8. **Define the following terms :**
i. **Attenuation constant (α)**
ii. **Phase constant (β)**
iii. **Loss tangent (tan θ)**

Answer

i. **Attenuation constant (α) :**
1. It is defined as a constant which indicates the rate at which the wave amplitude reduces as its propagates from one point to another.
2. It is measured in nepers per meter (Np/m) and can be expressed in dB/m.
3. An attenuation of 1 neper denotes a reduction to e^{-1} of the original value whereas an increase of 1 neper indicates an increase by a factor of e.

$$1\text{ Np} = 20\log_{10} e = 8.686\text{ dB}$$

ii. **Phase constant (β) :** The quantity β is a measure of the phase shift per unit length in radians per meter and is called the phase constant or wave number.

$$u = \frac{\omega}{\beta}, \quad \lambda = \frac{2\pi}{\beta}$$

where, u = Wave velocity

iii. **Loss tangent (tan θ) :** $\tan\theta$ or θ may be used to determine how lossy a medium is. A medium is said to be a good (lossless or perfect) dielectric if $\tan\theta$ is very small ($\sigma < \omega\epsilon$) or a good conductor if $\tan\theta$ is very large ($\sigma >> \omega\epsilon$).

$$\tan\theta = \sigma/\omega\epsilon$$

where, $\tan\theta$ = Loss tangent

θ = Loss angle of the medium.

Que 5.9. **A uniform plane wave propagating in a medium has $\vec{E} = 2e^{-\alpha z}\sin(10^8 t - \beta z)\ \hat{a}_y$ V/m. If a medium is characterized by $\varepsilon_r = 1$, $\mu_r = 20$ and $\sigma = 3$ S/m, determine α, β and $\vec{H}$.**

Answer

1. We need to determine the loss tangent to be able to tell whether the medium is a lossy dielectric or a good conductor.

$$\frac{\sigma}{\omega\epsilon} = \frac{3}{10^8 \times 1 \times \frac{10^{-9}}{36\pi}} = 3393 >> 1$$

Showing that the medium may be regarded as a good conductor at the frequency of operation.

2. Hence,
$$\alpha = \beta = \sqrt{\frac{\mu\omega\sigma}{2}} = \left[\frac{4\pi\times10^{-7}\times20(10^8)(3)}{2}\right]^{1/2}$$
$$= 61.4 \text{ Np/m}$$

3. Also,
$$|\eta| = \sqrt{\frac{\mu\omega}{\sigma}} = \left[\frac{4\pi\times10^{-7}\times20(10^8)}{3}\right]^{1/2}$$
$$= \sqrt{\frac{800\pi}{3}}$$
$$\tan 2\theta_\eta = \frac{\sigma}{\omega\epsilon} = 3393 \rightarrow \theta_\eta = 45° = \frac{\pi}{4}$$

4. Hence,
$$\vec{H} = H_0 e^{-\alpha z}\sin\left(\omega t - \beta z - \frac{\pi}{4}\right)\hat{a}_H$$

where,
$$\hat{a}_H = \hat{a}_k \times \hat{a}_E = \hat{a}_z \times \hat{a}_y = -\hat{a}_z$$

and
$$H_0 = \frac{E_0}{|\eta|} = 2\sqrt{\frac{3}{800\pi}} = 69.1\times10^{-3}$$

Thus,
$$\vec{H} = -69.1\,e^{-61.4z}\sin\left(10^8 t - 61.42z - \frac{\pi}{4}\right)(\hat{a}_x) \text{ mA/m}$$

PART-5

Plane Waves in Lossless Dielectrics, Plane Waves in Free Space, Plane Waves in Good Conductors.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.10. **Discuss the propagation of plane waves in**
i. Lossless dielectrics
ii. Free space
iii. Good conductors.

OR

Derive all expressions of an EM Wave like attenuation constant, phase constant and intrinsic impedance when it is propagating through a free space.

AKTU 2019-20, Marks 10

Answer

i. **Lossless dielectric :**

1. In a lossless dielectric, $\sigma << \omega\epsilon$. In this case $\sigma = 0$, $\epsilon = \epsilon_0\epsilon_r$, $\mu = \mu_0\mu_r$.
2. The equations for α, β, η are

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\epsilon}\right]^2} - 1\right]} \quad ...(5.10.1)$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\epsilon}\right]^2} + 1\right]} \quad ...(5.10.2)$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \quad ...(5.10.3)$$

3. So, we get,
$$\alpha = 0, \ \beta = \omega\sqrt{\mu\epsilon}$$
$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}, \ \lambda = \frac{2\pi}{\beta}, \text{ also } \eta = \sqrt{\frac{\mu}{\epsilon}}\angle 0°$$

4. Thus, $\vec{E}$ and $\vec{H}$ are in time phase with each other.

ii. **Free space :**

1. Propagation of plane waves in free space comprise of $\sigma = 0$, $\epsilon = \epsilon_0$ and $\mu = \mu_0$.

2. Thus, replacing $\in$ by $\in_0$, μ by μ_0 and putting $\sigma = 0$ in eq. (5.10.1) and (5.10.2), we get

$$\alpha = 0,\ \beta = \omega\sqrt{\mu_0 \in_0} = \frac{\omega}{c}$$

$$u = \frac{1}{\sqrt{\mu_0 \in_0}} = c,\ \lambda = \frac{2\pi}{\beta}$$

3. If we substitute the conditions $\sigma = 0$, $\in = \in_0$, $\mu = \mu_0$ and $\theta_n = 0$, $\eta = \eta_0$, in the eq. (5.10.3)

$$\eta = \frac{\sqrt{\mu/\in}}{\left[1+\left(\frac{\sigma}{\omega \in}\right)^2\right]^{1/4}},\ \tan 2\theta_\eta = \frac{\sigma}{\omega \in} \quad ...(5.10.4)$$

then we obtain, $\eta_0 = \sqrt{\frac{\mu_0}{\in_0}} = 120\pi = 377\ \Omega$

iii. Good conductor :

1. A perfect or good conductor is that in which $\sigma >> \omega\in$ so, $\frac{\sigma}{\omega \in} >> 1$
2. Plane waves in good conductor comprise of $\sigma = \infty$, $\in = \in_0$ and $\mu = \mu_0\, \mu_r$.
3. Putting the above conditions in eq. (5.10.1), (5.10.2) and (5.10.3), we get

$$\alpha = \beta = \sqrt{\frac{\omega\,\mu\,\sigma}{2}} = \sqrt{\pi f \mu \sigma}$$

$$u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}},\ \lambda = \frac{2\pi}{\beta}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\,\mu}{\sigma}}\ \angle 45°$$

Que 5.11. **Explain depth of penetration and skin effect.**

Answer

Depth of penetration and skin effect :

1. The depth of penetration is defined as that depth at which waves attenuates to $1/e$ of its original amplitude. Depth of penetration is also called skin depth.
2. It is a measure of depth to which an electromagnetic wave can penetrate the medium.
3. The depth of penetration, $\delta = \frac{1}{\alpha}$

   where, α = Attenuation constant.
4. The phenomenon whereby field intensity in a conductor rapidly decreases is known as skin effect.
5. It is the tendency of charges to migrate from bulk of conducting material to the surface, resulting in higher resistance.

Que 5.12. **Explain Skin effect. Derive the expression for α and β in a conducting medium.**

AKTU 2017-18, Marks 07

Answer

A. **Skin effect :** Refer Q. 5.11, Page 5–14B, Unit-5.

B. **Expression for α and β in a conducting medium :**

1. γ is called the propagation constant of the medium. It is given as

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\in)} \quad ...(5.12.1)$$

2. Since γ is a complex quantity, we may let

$$\gamma = \alpha + j\beta$$
$$\gamma^2 = (\alpha + j\beta)^2 = \alpha^2 - \beta^2 + 2j\alpha\beta \quad ...(5.12.2)$$

3. From eq. (5.12.1) and eq. (5.12.2) we, have

$$\alpha^2 - \beta^2 + j(2\alpha\beta) = j\omega\mu\sigma - \omega^2\mu\in$$

Comparing real and imaginary parts we have

$$\alpha^2 - \beta^2 = -\omega^2\mu\in \quad ...(5.12.3)$$

$$\mu\omega\sigma = 2\alpha\beta \text{ or } \alpha\beta = \frac{\mu\omega\sigma}{2}$$

$$\alpha = \frac{\mu\omega\sigma}{2\beta} \text{ and } \beta = \frac{\mu\omega\sigma}{2\alpha} \quad ...(5.12.4)$$

4. From eq. (5.12.4) and eq. (5.12.3), we have

$$\alpha^2 - \frac{\mu^2\omega^2\sigma^2}{4\alpha^2} = -\omega^2\mu\in$$

$$4\alpha^4 - \mu^2\omega^2\sigma^2 = -4\alpha^2\omega^2\mu\in$$

$$4\alpha^4 + \omega^2\mu\in(4\alpha^2) - \mu^2\omega^2\sigma^2 = 0$$

$$\alpha^4 + \omega^2\mu\in(\alpha^2) - \frac{\mu^2\omega^2\sigma^2}{4} = 0$$

$$\alpha^4 + \omega^2\mu\in(\alpha^2) = \left(\frac{\omega\mu\sigma}{2}\right)^2 \quad ...(5.12.5)$$

5. Adding the term $\left(\frac{\omega^2\mu\in}{2}\right)^2$ on both sides of eq. (5.12.5), we get

$$\alpha^4 + \alpha^2\omega^2\mu\in + \left(\frac{\omega^2\mu\in}{2}\right)^2 = \frac{\omega^2\mu^2\sigma^2}{4} + \left(\frac{\omega^2\mu\in}{2}\right)^2$$

$$\left(\alpha^2 + \frac{\omega^2\mu\in}{2}\right)^2 = \frac{\omega^2\mu^2\sigma^2}{4} + \frac{\omega^4\mu^2\in^2}{4}$$

$$\left[\alpha^2 + \frac{\omega^2\mu\in}{2}\right]^2 = \frac{\omega^4\mu^2\in^2}{4}\left[1 + \frac{\sigma^2}{\omega^2\in^2}\right]$$

$$\alpha^2 + \frac{\omega^2\mu\in}{2} = \frac{\omega^2\mu\in}{2}\sqrt{1 + \frac{\sigma^2}{\omega^2\in^2}}$$

$$\alpha^2 = \frac{\omega^2\mu\in}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\in^2}}-1\right] \quad ...(5.12.6)$$

$$\alpha = \sqrt{\frac{\omega^2\mu\in}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\in^2}}-1\right]}$$

$$\alpha = \omega\sqrt{\frac{\mu\in}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2}-1\right]} \text{ neper/meter}$$

6. Now substituting eq. (5.12.6) in eq. (5.12.3), we get

$$\frac{\omega^2\mu\in}{2}\sqrt{1+\frac{\sigma^2}{\omega^2\in^2}}-\frac{\omega^2\mu\in}{2}-\beta^2 = -\omega^2\mu\in$$

$$\beta^2 = \frac{\omega^2\mu\in}{2}\sqrt{1+\frac{\sigma^2}{\omega^2\in^2}}+\frac{\omega^2\mu\in}{2}$$

$$\beta = \omega\sqrt{\frac{\mu\in}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2}+1\right)}$$

Que 5.13. **A 10 GHz plane wave travelling in free space has an amplitude of E_x = 10 V/m. Find v, η, β, λ and the amplitude of H.**

Answer

Given : E_x = 10 V/m and f = 10 GHz
To Find : v, η, β, γ, H.

1. $$c = \lambda f$$
$$\lambda = \left(\frac{3\times10^8}{10\times10^9}\right) = 0.03 \text{ m}$$

2. $$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{0.03} = 209.43 \text{ rad/sec}$$

3. $$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\pi\times10^{-7}}{8.854\times10^{-12}}} = 376.7\ \Omega$$

4. $$v = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8 \text{ m/ sec}$$

5. $$H = \frac{|E|}{120\pi} = 0.02652 \text{ V/m}$$

Que 5.14. **A plane electromagnetic wave propagating in z direction in a dielectric medium of permittivity ε_r = 5, the electric field is in x-direction and has an RMS value 0.1 V/m. What is the direction and magnitude of magnetic field ? Also calculate the frequency of wave.**

AKTU 2018-19, Marks 10

Answer

Given : $\varepsilon_r = 5$, E_{rms} = 0.1 V/m
To Find : Direction of magnetic field, $\vec{H}$, magnitude of magnetic field, H and frequency of wave, f.

Direction of magnetic field :

$$\vec{E}\times\vec{H} = \text{Direction of propagation}$$
$$\hat{x}\times\vec{H} = \hat{z}$$
$$\vec{H} = \hat{y}$$

Direction of magnetic field = y axis.

Magnitude of magnetic field :

1. As we know, $$\frac{E}{H} = \eta$$
$$H = \frac{E}{\eta}$$

2. Here $$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$$
$$= \frac{120\pi}{\sqrt{5}} = 168.60\ \Omega$$

3. So, $$H = \frac{E}{168.60} = \frac{0.1\sqrt{2}}{168.60}$$
$$= 8.39\times10^{-4} \text{ A/m}$$

4. Velocity of propagation,
$$u = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{3\times10^8}{\sqrt{\mu_r\varepsilon_r}} = \frac{3\times10^8}{\sqrt{1\times5}} = 1.34\times10^8 \text{ m/s}$$

5. We know that, $u = f\lambda$
Assuming, $\lambda = 5$ m
$$\therefore \quad f = \frac{u}{\lambda} = 26.83 \text{ MHz}$$

Que 5.15. **A uniform plane wave propagating in good conductor. If the magnetic field intensity is given by**

$$\vec{H} = 0.1\,e^{-15z}\cos(2\pi\times10^8 t - 15z)\,\hat{a}_x \text{ A/m},$$

determine the conductivity and corresponding component of E field. Also calculate the average power loss in a block of unit area and thickness t.

AKTU 2017-18, Marks 07

Answer

1. Given, $H = 0.1e^{-15z} \cos (2\pi \times 10^8 t - 15z)\ \hat{a}_x$

On comparing with the general equation for $\vec{H}$ we get,

$$H_0 = 0.1,\ \alpha = 15,\ \beta = 15,\ w = 2\pi \times 10^8 \text{ rad/s}$$

2. Also, $\alpha = \beta = \sqrt{\pi\mu f \sigma}$

$$\sigma = \frac{\alpha^2}{\pi\mu f} = \frac{225}{\pi \times 4\pi \times 10^{-7} \times 10^8} = 0.57 \text{ S/m}$$

and $t = \frac{1}{\alpha} = \frac{1}{15} = 0.067$ m

3. Assume $\varepsilon = \varepsilon_0$

$$\frac{\sigma}{\omega\varepsilon} = \frac{0.57}{2\pi \times 10^8 \times 8.85 \times 10^{-12}} = \frac{57}{2\pi \times 8.85} = 1.025$$

$$\theta_n = \frac{1}{2}\tan^{-1}(1.025) = 22.85°$$

$$|\eta| = \frac{337}{1.20} = 315\ \Omega$$

4. $\vec{E}(z, t) = -31.5e^{-15z} \cos (2\pi \times 10^8 t - 15z + \theta\eta)\ \hat{a}_y$ V/m

5. Average power can be given as :

$$P_{avg} = \frac{1}{2}[E \times H^*] = \frac{1}{2} \times 0.1 \times 31.5e^{-30z} \cos 22.85°$$
$$= 1.45\, e^{-30z} \text{ W/m}^2$$

At $z = 0$, $P_{avg}(0) = 1.45$

At $z = \delta$, $P_{avg}(\delta) = 1.45e^{-2} = 1.45 \times 0.135$

6. Average power loss is given by

$$\text{Power loss} = P_{avg}(0) - P_{avg}(\delta) = 1.45\,[1 - 0.135] = 1.254 \text{ W}$$

Que 5.16. **Derive the wave equation for electric field.**

OR

Write wave equations for conducting medium.

Answer

1. The Maxwell's equations for fields are given as

$$\nabla \cdot \vec{B} = 0 \quad \text{...(5.16.1)}$$

$$\nabla \cdot \vec{D} = \rho_v \quad \text{...(5.16.2)}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{...(5.16.3)}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t} \quad \text{...(5.16.4)}$$

2. Concerning the characteristics of medium, three equations are given as

$$\vec{D} = \epsilon \vec{E} \quad \text{...(5.16.5)}$$

$$\vec{B} = \mu \vec{H} \quad \text{...(5.16.6)}$$

$$\vec{J} = \sigma \vec{E} \quad \text{...(5.16.7)}$$

Wave equation for electric field (free space) :

1. Since ϵ and μ are independent of time so differentiating eq. (5.16.5) and (5.16.6) *w.r.t.* time

we get,

$$\frac{\partial \vec{D}}{\partial t} = \epsilon \frac{\partial \vec{E}}{\partial t} \quad \text{...(5.16.8)}$$

$$\frac{\partial \vec{B}}{\partial t} = \mu \frac{\partial \vec{H}}{\partial t} \quad \text{...(5.16.9)}$$

2. For free space, $\sigma = 0$

Therefore, eq. (5.16.4) will become

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \quad \text{...(5.16.10)}$$

3. From eq. (5.16.8) and (5.16.10), we get

$$\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} \quad \text{...(5.16.11)}$$

Differentiating eq. (5.16.11) *w.r.t.* time, we get

$$\nabla \times \frac{\partial \vec{H}}{\partial t} = \epsilon \frac{\partial^2 \vec{E}}{\partial t}$$

Taking curl on both sides of eq. (5.16.3), we get

$$\nabla \times (\nabla \times \vec{E}) = -\nabla \times \frac{\partial \vec{B}}{\partial t} \quad \text{...(5.16.12)}$$

4. Using eq. (5.16.9), we get $\nabla \times (\nabla \times \vec{E}) = -\mu \nabla \times \frac{\partial \vec{H}}{\partial t}$ ...(5.16.13)

Putting value of $\nabla \times \frac{\partial \vec{H}}{\partial t}$ from eq. (5.16.11) in eq. (5.16.13), we get

$$\nabla \times (\nabla \times \vec{E}) = -\mu\, \epsilon \times \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{...(5.16.14)}$$

5. From the identity equation, we have

$$\nabla \times (\nabla \times \vec{E}) = \nabla \times (\nabla \times \vec{E}) - \nabla^2 \vec{E} \quad \text{...(5.16.15)}$$

Replacing $\nabla \times (\nabla \times \vec{E})$ from eq. (5.16.15) into eq. (5.16.14)

$$\nabla \times (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad ...(5.16.16)$$

6 For free space, eq. (5.16.2) will give

$$\nabla \cdot \vec{D} = 0$$

or, $$\nabla \cdot \epsilon \vec{E} = 0$$

$$\nabla \cdot \vec{E} = 0 \quad ...(5.16.17)$$

7. From eq. (5.16.17) and (5.16.6), we get

$$\nabla(0) - \nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

or, $$\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad ...(5.16.18)$$

Similarly, we can obtain

$$\nabla^2 \vec{H} = \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} \quad ...(5.16.19)$$

Eq. (5.16.18) and (5.16.19) are known as wave equations.

8. For free space $\mu = \mu_o$ and $\epsilon = \epsilon_o$.
Then wave equations for free space will be

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad ...(5.16.20)$$

$$\nabla^2 \vec{H} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} \quad ...(5.16.21)$$

Wave equation (for conducting or lossy medium) :

1. Taking eq. (5.16.4),

$$\nabla \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}$$

or, $$\nabla \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad (\because \vec{D} = \epsilon \vec{E}) \quad ...(5.16.22)$$

2. Taking eq. (5.16.3), $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial(\mu \vec{H})}{\partial t} \quad (\because \vec{B} = \mu \vec{H})$

or, $$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad ...(5.16.23)$$

Taking curl on both sides of eq. (5.16.23), we get

$$\nabla \times (\nabla \times \vec{E}) = -\mu \nabla \times \frac{\partial \vec{H}}{\partial t}$$

Putting $\nabla \times \frac{\partial \vec{H}}{\partial t}$ from eq. (5.16.22), we get

$$\nabla \times (\nabla \times \vec{E}) = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} - \mu\sigma \frac{\partial \vec{E}}{\partial t} \quad ...(5.16.24)$$

or, $$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} - \mu\sigma \frac{\partial \vec{E}}{t} \quad ...(5.16.25)$$

3. Since, we have $\nabla \cdot \vec{D} = 0$

$$\nabla \cdot \epsilon \vec{E} = 0$$

$$\nabla \cdot \vec{E} = 0$$

4. Therefore, from eq. (5.16.25)

$$0 - \nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} - \mu\sigma \frac{\partial \vec{E}}{\partial t}$$

or $$-\nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} - \mu\sigma \frac{\partial \vec{E}}{\partial t}$$

or, $$\nabla^2 \vec{E} - \mu \left\{ \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \sigma \frac{\partial \vec{E}}{\partial t} \right\} = 0 \quad ...(5.16.26)$$

Similarly, equation for $\vec{H}$ can also be obtained as

$$\nabla^2 \vec{H} - \mu \left\{ \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} + \sigma \frac{\partial \vec{H}}{\partial t} \right\} = 0 \quad ...(5.16.27)$$

Eq. (5.16.26) and (5.16.27) are the wave equations for conducting media.

PART-6

Power and the Pointing Vector.

Questions-Answers
Long Answer Type and Medium Answer Type Questions

Que 5.17. **Explain Poynting vector. Derive an expression of Poynting theorem for EM wave. Also explain the significance of each term of the expression.** **AKTU 2018-19, Marks 07**

OR

State Poynting theorem. Derive the mathematical expression for Poynting theorem. **AKTU 2017-18, Marks 07**

OR

Derive the mathematical equation for poynting vector.

AKTU 2021-22, Marks 10

Answer

1. Energy can be transported from one point to another point by means of EM waves. The rate of such energy transportation can be obtained from Maxwell's equations :

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad ...(5.17.1)$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad ...(5.17.2)$$

2. Taking a dot product on both sides of eq. (5.17.2) with $\vec{E}$ gives

$$\vec{E} \cdot (\nabla \times \vec{H}) = \sigma E^2 + \vec{E} \cdot \epsilon \frac{\partial \vec{E}}{\partial t} \quad ...(5.17.3)$$

3. For any vector fields $\vec{A}$ and $\vec{B}$

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

Applying this vector identity to eq. (5.17.3) (letting $\vec{A} = \vec{H}$ and $\vec{B} = \vec{E}$) gives

$$\vec{H} \cdot (\nabla \times \vec{E}) + \nabla \cdot (\vec{H} \times \vec{E}) = \sigma E^2 + \vec{E} \cdot \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$= \sigma E^2 + \frac{1}{2} \epsilon \frac{\partial E^2}{\partial t} \quad ...(5.17.4)$$

4. Taking a dot product on both sides of eq. (5.17.1) with $\vec{H}$,

$$\vec{H} \cdot (\nabla \times \vec{E}) = \vec{H} \cdot \left(-\mu \frac{\partial \vec{H}}{\partial t}\right) = \frac{-\mu}{2} \frac{\partial}{\partial t} (\vec{H} \cdot \vec{H}) \quad ...(5.17.5)$$

5. Thus eq. (5.17.4) becomes,

$$\frac{-\mu}{2} \frac{\partial H^2}{\partial t} - \nabla \cdot (\vec{E} \times \vec{H}) = \sigma E^2 + \frac{1}{2} \epsilon \frac{\partial E^2}{\partial t}$$

Rearranging terms and taking volume integral of both sides,

$$\int_v \nabla \cdot (\vec{E} \times \vec{H})\, dv = -\frac{\partial}{\partial t} \int_v \left[\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2\right] dv - \int_v \sigma E^2 dv \quad ...(5.17.6)$$

Applying the divergence theorem to the left-hand side gives

$$\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{S} = -\frac{\partial}{\partial t} \int_v \left[\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2\right] dv - \int_v \sigma E^2 dv \quad ...(5.17.7)$$

$$\left(\begin{array}{c}\text{Total power leaving}\\ \text{the volume}\end{array}\right) = \left(\begin{array}{c}\text{Rate of decrease in}\\ \text{energy stored in electric}\\ \text{and magnetic fields}\end{array}\right) - \left(\begin{array}{c}\text{Ohmic power}\\ \text{dissipated}\end{array}\right)$$

6. Eq. (5.17.7) is referred to as Poynting's theorem. The quantity $\vec{E} \times \vec{H}$ is known as the Poynting vector P, measured in watts per square meter (W/m^2) ; *i.e.*,

$$P = \vec{E} \times \vec{H}$$

It represents the instantaneous power density vector associated with the EM field at a given point.

7. The integration of Poynting vector over any closed surface gives the net power flowing out of that surface.
8. Poynting theorem states that the net power flowing out of a given volume v is equal to the time rate of decrease in the energy stored within v minus the Ohmic losses.

PART-7

Reflection of a Plain Wave in a Normal Incidence.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.18. **Explain the reflection of plane wave for the normal incidence. Discuss about reflection and transmission coefficient for E and H.** **AKTU 2017-18, Marks 07**

Answer

1. If load impedance (Z_L) is not equal to characteristic impedance (Z_0) then mismatch occurs. In this case the part of the wave gets absorbed by the

load while the rest part is reflected back to the source. Thus, due to mismatch in impedance the reflection occurs.

2. Suppose that a plane wave propagating along +z direction incident normally on the boundary $z = 0$ between medium 1 ($z < 0$) characterized by $\sigma_1, \epsilon_1, \mu_1$ and medium 2 ($z > 0$) characterized by $\sigma_2, \epsilon_2, \mu_2$ as shown in Fig. 5.18.1.

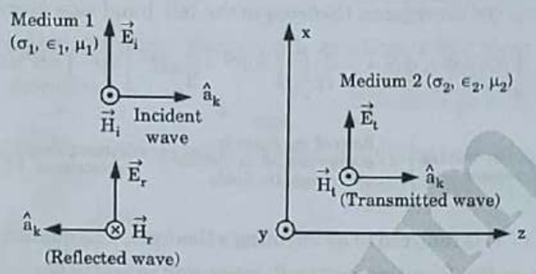

Fig. 5.18.1.

3. **Incident wave :** $(\vec{E}_i, \vec{H}_i)$ is traveling along $+\hat{a}_z$ in medium 1. Assume that

$$\vec{E}_{is}(z) = E_{i0}e^{-\gamma_1 z}\hat{a}_x \quad ...(5.18.1)$$

then $$\vec{H}_{is}(z) = H_{i0}e^{-\gamma_1 z}\hat{a}_y = \frac{E_{i0}}{\eta_1}e^{-\gamma_1 z}\hat{a}_y \quad ...(5.18.2)$$

4. **Reflected wave :** $(\vec{E}_r, \vec{H}_r)$ is traveling along $-\hat{a}_z$ in medium 1. If

$$\vec{E}_{rs}(z) = E_{r0}e^{-\gamma_1 z}\hat{a}_x \quad ...(5.18.3)$$

then, $$\vec{H}_{rs}(z) = H_{r0}e^{-\gamma_1 z}(-\hat{a}_y) = -\frac{E_{r0}}{\eta_1}e^{-\gamma_1 z}\hat{a}_y \quad ...(5.18.4)$$

5. **Transmitted wave :** $(\vec{E}_t, \vec{H}_t)$ is traveling wave along $+\hat{a}_z$ in medium 2.

If $$\vec{E}_{ts}(z) = E_{t0}e^{-\gamma_2 z}\hat{a}_x \quad ...(5.18.5)$$

then $$\vec{H}_{ts}(z) = H_{t0}e^{-\gamma_2 z}\hat{a}_y = \frac{E_{t0}}{\eta_2}e^{-\gamma_2 z}\hat{a}_y \quad ...(5.18.6)$$

6. In eq. (5.18.1) to (5.18.6), E_{i0}, E_{r0}, E_{t0} are respectively, the magnitude of the incident, reflected and transmitted electric fields at $z = 0$.
As seen from Fig. 5.18.1, the total field in medium 1 comprises both the incident and reflected fields, where as medium 2 has only transmitted field, *i.e.*,

$$\vec{E}_1 = \vec{E}_i + \vec{E}_r \quad , \quad \vec{H}_1 = \vec{H}_i + \vec{H}_r$$

$$\vec{E}_2 = \vec{E}_t \quad , \quad \vec{H}_2 = \vec{H}_t$$

8. At the interface $z = 0$, the boundary conditions require that the tangential components of $\vec{E}$ and $\vec{H}$ must be continuous.

9. At $z = 0$, $\vec{E}_{1\tan} = \vec{E}_{2\tan}$ and $\vec{H}_{1\tan} = \vec{H}_{2\tan}$ imply that

$$\vec{E}_i(0) + \vec{E}_r(0) = \vec{E}_{t0} \quad \Rightarrow E_{i0} + E_{r0} = E_{t0} \quad ...(5.18.7)$$

$$\vec{H}_i(0) + \vec{H}_r(0) = \vec{H}_t(0) \Rightarrow \frac{1}{\eta_1}(E_{i0} - E_{r0}) = \frac{E_{t0}}{\eta_2} \quad ...(5.18.8)$$

10. From eq. (5.18.7) and (5.18.8),

$$E_{r0} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}E_{i0} \quad ...(5.18.9)$$

and $$E_{t0} = \frac{2\eta_2}{\eta_2 + \eta_1}E_{i0}$$

$\therefore$ Reflection coefficient, $\Gamma = \dfrac{E_{r0}}{E_{i0}} = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ ...(5.18.10)

and transmission coefficient, $\tau = \dfrac{E_{t0}}{E_{i0}} = \dfrac{2\eta_2}{\eta_2 + \eta_1}$ ...(5.18.11)

Que 5.19. **Explain the reflection of a plane wave at oblique incidence. Calculate reflection and transmission coefficients.**

Answer

Reflection of a plane wave at oblique incidence :

1. Reflection and transmission of a wave depends on :
i. The type of polarization of wave.
ii. The medium of boundary.
2. Generally, there are two types of polarization :
a. **Parallel polarization :**
1. Fig. 5.19.1 shows parallel polarization, where $\vec{E}$ field lies in the x-z plane, the plane of incidence.
2. Hence, in medium 1, we have both incident and reflected fields given by

$$\vec{E}_{is} = E_{i0}(\cos\theta_i\,\hat{a}_x - \sin\theta_i\,\hat{a}_z)\,e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \quad ...(5.19.1)$$

$$\vec{H}_{is} = \frac{E_{i0}}{\eta_1}e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\hat{a}_y \quad ...(5.19.2)$$

$$\vec{E}_{rs} = E_{r0}(\cos\theta_r\,\hat{a}_z + \sin\theta_r\,\hat{a}_z)e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \quad ...(5.19.3)$$

$$\vec{H}_{rs} = -\frac{E_{r0}}{\eta}e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\hat{a}_y \quad ...(5.19.4)$$

where, $\beta_1 = \omega\sqrt{\mu_1 \epsilon_1}$

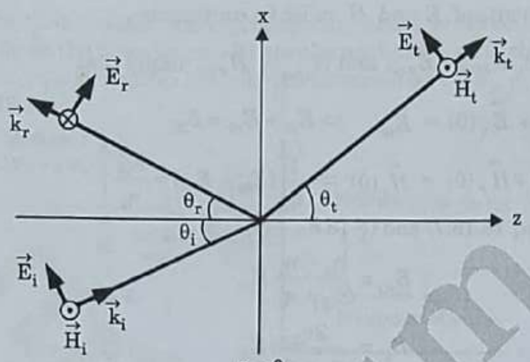

Fig. 5.19.1.

3. The transmitted fields exist in medium 2 and are given by

$$\vec{E}_{ts} = E_{t0}(\cos\theta_t\,\hat{a}_x - \sin\theta_t\,\hat{a}_z)e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \quad ...(5.19.5)$$

$$\vec{H}_{ts} = \frac{E_{t0}}{\eta_2}e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}\hat{a}_y \quad ...(5.19.6)$$

where $\beta_2 = \omega\sqrt{\mu_2 \epsilon_2}$

4. Requiring that $\theta_r = \theta_i$ and that the tangential components of $\vec{E}$ and $\vec{H}$ be continuous at the boundary $z = 0$, we obtain

$$(E_{i0} + E_{r0})\cos\theta_i = E_{t0}\cos\theta_t \quad ...(5.19.7)$$

$$\frac{1}{\eta_1}(E_{i0} - E_{r0}) = \frac{1}{\eta_2}E_{t0} \quad ...(5.19.8)$$

5. Now expressing E_{r0} and E_{t0} in terms of E_{i0}, we get reflection coefficient,

$$\Gamma_{||} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \quad ...(5.19.9)$$

and transmission coefficient,

$$\tau_{||} = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \quad ...(5.19.10)$$

b. Perpendicular polarization :

1. Fig. 5.19.2 shows perpendicular polarization where the $\vec{E}$ field is perpendicular to the plane of incidence (*x*-*z* plane). This may also be viewed as the case in which $\vec{H}$ field is parallel to the plane of incidence.

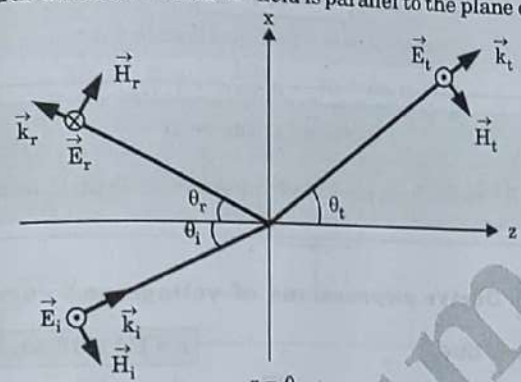

Fig. 5.19.2.

2. The incident and reflected fields in medium 1 are given by

$$\vec{E}_{is} = E_{i0}e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\hat{a}_y \quad ...(5.19.11)$$

$$\vec{H}_{is} = \frac{E_{i0}}{\eta_1}(-\cos\theta_i\,\hat{a}_x + \sin\theta_i\,\hat{a}_z)e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \quad ...(5.19.12)$$

$$\vec{E}_{rs} = E_{r0}\,e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\hat{a}_y \quad ...(5.19.13)$$

$$\vec{H}_{rs} = \frac{E_{r0}}{\eta_1}(\cos\theta_r\,\hat{a}_x + \sin\theta_r\,\hat{a}_z)e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \quad ...(5.19.14)$$

3. The transmitted fields in medium 2 are given by

$$\vec{E}_{ts} = E_{t0}\,e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}\hat{a}_y \quad ...(5.19.15)$$

$$\vec{H}_{ts} = \frac{E_{t0}}{\eta_2}(-\cos\theta_t\,\hat{a}_x + \sin\theta_t\,\hat{a}_z)e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \quad ...(5.19.16)$$

4. Requiring that the tangential components of $\vec{E}$ and $\vec{H}$ be continuous at $z = 0$ and setting θ_r equal to θ_i, we get

$$E_{i0} + E_{r0} = E_{t0} \quad ...(5.19.17)$$

$$\frac{1}{\eta_1}(E_{i0} - E_{r0})\cos\theta_i = \frac{1}{\eta_2}E_{t0}\cos\theta_t \quad ...(5.19.18)$$

5. Expressing E_{r0} and E_{t0} in terms of E_{i0}, we get

$$\Gamma_{\perp} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \quad ...(5.19.19)$$

and $$\tau_{\perp} = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \quad ...(5.19.20)$$

PART-8

Transmission Lines and Smith Chart.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.20. Derive expressions of voltage and current in a transmission line. **AKTU 2019-20, Marks 10**

OR

Derive a general expression of voltage and current of a transmission line. **AKTU 2018-19, Marks 07**

OR

What is transmission line ? Derive all the supporting mathematical equations of the transmission line. **AKTU 2021-22, Marks 10**

Answer

A. **Transmission line :** A transmission line is a means of transfer of information from one point to another. Usually it consists of two conductors. It is used to connect a source to a load. The source may be a transmitter and the load may be a receiver.

B. **Expression of voltage and current in a transmission line :**

1. By applying Kirchhoff's voltage law to the outer loop of the circuit in Fig. 5.20.1, we obtain

$$V(z,t) = R\,\Delta z I(z,t) + L\,\Delta z\frac{\partial I(z,t)}{\partial t} + V(z+\Delta z, t)$$

or

$$-\frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t} \quad ...(5.20.1)$$

2. Taking the limit of eq. (5.20.1) as $\Delta z \to 0$ leads to

$$-\frac{\partial V(z,t)}{\partial z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t} \quad ...(5.20.2)$$

3. Similarly, applying Kirchhoff's current law to the main node of the circuit in Fig. 5.20.1 gives

$$I(z,t) = I(z+\Delta z, t) + \Delta I$$

$$= I(z+\Delta z,t) + G\,\Delta z\,V(z+\Delta z,t) + C\,\Delta z\frac{\partial V(z+\Delta z,t)}{\partial t}$$

or

$$-\frac{I(z+\Delta z,t) - I(z,t)}{\Delta z} = G\,V(z+\Delta z,t) + C\frac{\partial V(z+\Delta z,t)}{\partial t} \quad ...(5.20.3)$$

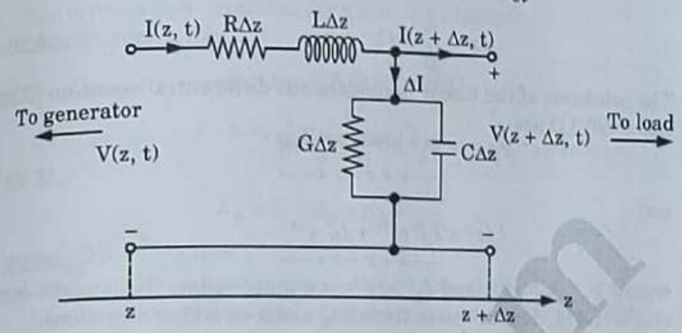

Fig. 5.20.1. An L-type equivalent circuit model of a two-conductor transmission line of differential length Δz.

4. As $\Delta z \to 0$, eq. (5.20.3) becomes

$$-\frac{\partial I(z,t)}{\partial z} = G\,V(z,t) + C\frac{\partial V(z,t)}{\partial t} \quad ...(5.20.4)$$

5. If we assume harmonic time dependence so that

$$V(z,t) = \text{Re}\,[V_s(z)\,e^{j\omega t}] \quad ...(5.20.5)$$

$$I(z,t) = \text{Re}\,[I_s(z)\,e^{j\omega t}]. \quad ...(5.20.6)$$

where $V_s(z)$ and $I_s(z)$ are the phasor forms of $V(z, t)$ and $I(z, t)$ respectively, eq. (5.20.2) and (5.20.4) become

$$-\frac{dV_s}{dz} = (R + j\omega L)\,I_s \quad ...(5.20.7)$$

$$-\frac{dI_s}{dz} = (G + j\omega C)\,V_s \quad ...(5.20.8)$$

6. Take the second derivative of V_s in eq. (5.20.7) and employ eq. (5.20.8) so that we obtain

$$\frac{d^2V_s}{dz^2} = (R + j\omega L)(G + j\omega C)\,V_s$$

or

$$\frac{d^2V_s}{dz^2} - \gamma^2 V_s = 0 \quad ...(5.20.9)$$

where, $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$ ...(5.20.10)

7. By taking the second derivative of I_s in eq (5.20.8) and employing eq. (5.20.7), we get

$$\frac{d^2I_s}{dz^2} - \gamma^2 I_s = 0 \quad ...(5.20.11)$$

8. Thus, γ in eq. (5.20.10) is the propagation constant, α is the attenuation, and β is the phase constant.

9. The wavelength λ and wave velocity u are, respectively, given by

$$\lambda = \frac{2\pi}{\beta} \quad ...(5.20.12)$$

$$u = \frac{\omega}{\beta} = f\lambda \quad ...(5.20.13)$$

10. The solutions of the linear homogeneous differential equation (5.20.9) and (5.20.11) are

$$V_s(z) = \underset{\longrightarrow +z}{V_0^+ e^{-\gamma z}} + \underset{-z \longleftarrow}{V_0^- e^{\gamma z}} \quad ...(5.20.14)$$

and

$$I_s(z) = \underset{\longrightarrow +z}{I_0^+ e^{-\gamma z}} + \underset{-z \longleftarrow}{I_0^- e^{\gamma z}} \quad ...(5.20.15)$$

where V_0^+, V_0^-, I_0^- and I_0^- are wave amplitudes; the + and – signs, respectively, denote waves traveling along $+z$ and $-z$ directions.

Que 5.21. **Define characteristic impedance.**

OR

Define propagation constant and characteristic impedance. Derive the boundary conditions for electric field between two dielectrics having different permittivity interfaces.

AKTU 2017-18, Marks 07

Answer

A. **Propagation constant :** The measure of the change in amplitude and phase unit distance is called the propagation constant.

B. **Characteristic impedance :**

1. The characteristic impedance Z_0 of the line is the ratio of the positively traveling voltage wave to the current wave at any point on the line.
2. The characteristic impedance is given by

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C} \quad ...(5.21.1)$$

or

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_0 + jX_0 \quad ...(5.21.2)$$

where, R_0 and X_0 are the real and imaginary parts of Z_0.

C. **Derivation :** Refer Q. 2.26, Page 2–29B, Unit-2.

Que 5.22. **Discuss the concept of lossless and distortionless line.**

Answer

i. **Lossless line :**

1. A transmission line is said to be lossless if the conductors of the line are perfect ($\sigma_c \approx \infty$) and the dielectric medium separating them is lossless ($\sigma = 0$).

2. For such a line, when $\sigma_c = \infty$ and $\sigma = 0$.

$$R = 0 = G$$

This is necessary condition for a line to be lossless.

3. Thus for such a line,

$$\alpha = 0, \quad \gamma = j\beta = j\omega\sqrt{LC}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda$$

$$X_0 = 0, \quad Z_0 = R_0 = \sqrt{\frac{L}{C}}$$

ii. **Distortionless line :**

1. A distortionless line is one in which the attenuation constant α is frequency independent while the phase constant β is linearly dependent on frequency.
2. A distortionless line results if the line parameters are such that

$$\frac{R}{L} = \frac{G}{C}$$

3. Thus, for a distortionless line,

$$\gamma = \sqrt{RG\left(1 + \frac{j\omega L}{R}\right)\left(1 + \frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG}\left(1 + \frac{j\omega C}{G}\right) = \alpha + j\beta$$

or

$$\alpha = \sqrt{RG}, \quad \beta = \omega\sqrt{LC}$$

Showing that α does not depend on frequency, whereas β is linear function of frequency.

4. Also,

$$Z_0 = \sqrt{\frac{R(1 + j\omega L/R)}{G(1 + j\omega C/G)}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} = R_0 + jX_0$$

or

$$R_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}, \quad X_0 = 0$$

and

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda$$

Que 5.23. **A transmission line operating at 500 MHz has $Z_0 = 80\ \Omega$, $\alpha = 0.4$ NP/m, $\beta = 1.5$ rad/m. Find the line parameters R, L, G and C.**

AKTU 2017-18, Marks 07

Answer

1. Since Z_0 is real and $\alpha \neq 0$, this is a distortionless line.

$$Z_0 = \sqrt{R/G} \quad ...(5.23.1)$$

and, $$\frac{L}{R} = \frac{C}{G} \quad ...(5.23.2)$$

and, $$\alpha = \sqrt{RG} \quad ...(5.23.3)$$

and, $$\beta = \omega L\sqrt{G/R} \quad ...(5.23.4)$$

2. Multiplying eq. (5.23.1) and (5.23.3), $R = \alpha Z_0 = 0.04 \times 80 = 3.2\ \Omega/m$
3. Dividing eq. (5.23.3) by (1), $G = \alpha/Z_0 = 0.04/80 = 5 \times 10^{-4}\ \Omega/m$
4. Multiplying eq. (5.23.1) and (4), $L = \dfrac{\beta Z_0}{\omega} = \dfrac{1.5 \times 80}{2\pi \times 5 \times 10^8} = 38.2$ nH/m
5. From eq. (5.23.2), $C = \dfrac{LG}{R} = \dfrac{12}{\pi}10^{-8} \times \dfrac{0.04}{80} \times \dfrac{1}{0.04 \times 80} = 5.97$ pF/m

Que 5.24. **Derive an expression for characteristic impedance, input impedance of a transmission line.** **AKTU 2019-20, Marks 10**

Answer

A. Characteristic impedance : Refer Q. 5.21, Page 5–30B, Unit-5.

B. Input impedance :

1. Input impedance is defined as the ratio of voltage and current at the sending end.
2. Let the transmission line extend from $z = 0$ at the generator to $z = l$ at the load.

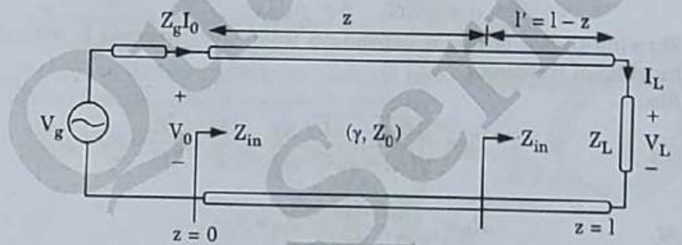

Fig. 5.24.1.

3. We know, $$V_s(z) = V_0^+e^{-\gamma z} + V_0^-e^{\gamma z} \quad ...(5.24.1)$$
$$I_s(z) = \frac{V_0^+e^{-\gamma z}}{Z_0} - \frac{V_0^-e^{\gamma z}}{Z_0} \quad ...(5.24.2)$$
4. If the conditions at the input be
$$V_0 = V(z = 0), I_0 = I(z = 0)$$
then, $$V_0^+ = \frac{1}{2}(V_0 + Z_0I_0) \text{ and } V_0^- = \frac{1}{2}(V_0 - Z_0I_0) \quad ...(5.24.3)$$
5. If the conditions at the load be
$$V_L = V(z = l), I_L = I(z = l)$$

then, $$V_0^+ = \frac{1}{2}(V_L + Z_0I_L)e^{\gamma l} \quad ...(5.24.4)$$
and $$V_0^- = \frac{1}{2}(V_L - Z_0I_L)e^{-\gamma l} \quad ...(5.24.5)$$

6. From eq. (5.24.1) and (5.24.2),
$$Z_{in} = \frac{V_S(z)}{I_S(z)} = \frac{Z_0(V_0^+ + V_0^-)}{V_0^+ - V_0^-} \quad ...(5.24.6)$$
7. We know that, $\dfrac{e^{\gamma l} + e^{-\gamma l}}{2} = \cosh \gamma l$, $\dfrac{e^{\gamma l} - e^{-\gamma l}}{2} = \sinh \gamma l$
$$\therefore \quad \tanh \gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}}$$
Substituting eq. (5.24.4) and eq. (5.24.5) into eq. (5.24.6) we get
$$Z_{in} = Z_0\left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}\right] \text{(lossy)} \quad ...(5.24.7)$$
8. For a lossless line, $\gamma = j\beta$, $\tanh j\beta l = j\tan\beta l$, and $Z_0 = R_0$, so eq. (5.24.7) becomes
$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan \gamma l}{Z_0 + jZ_L \tan \gamma l}\right] \text{(lossless)} \quad ...(5.24.8)$$
Showing that the input impedance varies periodically with distance l from the load.
9. When output impedance is short, *i.e.*, $Z_L = 0$,
then, $$Z_{in} = Z_{sc}\big|_{Z_L=0} = jZ_0 \tan \beta l$$

Que 5.25. **Derive the relation between reflection coefficient and voltage standing wave ratio (VSWR).**

Answer

Relation between reflection coefficient and voltage standing wave ratio :

1. If there is proper impedance matching at the load side, then the maximum energy will be transmitted to the load but if there is any mismatch in load impedance then some part of energy is reflected back towards the transmitting end. This reflected wave forms a standing wave of definite maxima and minima.
2. The reflection coefficient is the ratio of the reflection wave to the incident wave, that is
$$\Gamma = \frac{V_0^-e^{+\gamma l}}{V_0^+e^{-\gamma l}} \quad ...(5.25.1)$$

or $$|\Gamma| = \frac{|V_0^-|}{|V_0^+|} = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \{\because |e^{-\gamma l}| = |e^{\gamma l}| = 1\}$$

3. The voltage standing wave ratio is defined as

$$s = \text{VSWR} = \frac{\text{Maximum voltage or current}}{\text{Minimum voltage or current}}$$

$$s = \frac{|V_{max}|}{|V_{min}|} = \frac{|I_{max}|}{|I_{min}|}$$

4. The $|V_{max}| = |V_0^+| + |V_0^-|$

and $|V_{min}| = |V_0^+| - |V_0^-|$

5. Therefore, the voltage standing wave ratio is given by

$$s = \frac{|V_0^+| + |V_0^-|}{|V_0^+| - |V_0^-|} = \frac{|V_0^+|\left\{1 + \frac{|V_0^-|}{|V_0^+|}\right\}}{|V_0^+|\left\{1 - \frac{|V_0^-|}{|V_0^+|}\right\}}$$

or $$s = \frac{1 + \frac{|V_0^-|}{|V_0^+|}}{1 - \frac{|V_0^-|}{|V_0^+|}} \qquad ...(5.25.2)$$

6. From eq. (5.25.1) and (5.25.2), we get

$$\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad ...(5.25.3)$$

Que 5.26. **Relate short circuit, open circuit and characteristic of transmission line.**

Answer

1. If the line is open-circuited, $Z_L = \infty$

$$Z_{OC} = \lim_{Z_L \to \infty} Z_{in} = \lim_{Z_L \to \infty} Z_0 \left[\frac{1 + \frac{jZ_0}{Z_L}\tan\beta l}{\frac{Z_0}{Z_L} + j\tan\beta l}\right]$$

$$= \frac{Z_0}{j\tan\beta l} = -jZ_0 \cot\beta l \qquad ...(5.26.1)$$

2. If line is short-circuited, $Z_L = 0$

$$Z_{SC} = Z_{in}|_{Z_L=0} = Z_0\left[\frac{0 + jZ_0\tan\beta l}{Z_0}\right]$$

$$= jZ_0 \tan\beta l \qquad ...(5.26.2)$$

3. Using equation (5.26.1) and (5.26.2)

$$Z_{OC} \times Z_{SC} = (jZ_0 \tan\beta l)(-jZ_0 \cot\beta l)$$

$$Z_{OC} \times Z_{SC} = Z_0^2$$

4. Characteristic impedance

$$Z_0 = \sqrt{Z_{oc}\, Z_{sc}}$$

Que 5.27. **What is Smith chart ? Explain how it is constructed.**

OR

Explain uniform plane wave. Derive uniform plane waves in lossless dielectrics. What is skin effect ? Explain the smith chart in detail.

AKTU 2021-22, Marks 10

Answer

A. **Uniform plane wave :** A uniform plane wave is defined as the magnitude of the electric and magnetic fields. The electric and magnetic fields are orthogonal to the direction of propagation.

B. **Uniform plane wave in lossless dielectrics :** Refer Q. 5.10, Page 5–13B, Unit-5.

C. **Skin effect :** Refer Q. 5.11, Page 5–14B, Unit-5.

D. **Smith chart :**

1. It is a graphical method used for solving lossless transmission line problems.
2. It consists of a group of concentric circles, *i.e.*, circles corresponding to the constant resistance and conductance.
3. This chart comprises of the values of impedance and admittance in the normalized form.

Normalized impedance :

$$Z_n = \frac{Z}{\text{characterstics impedance}} = \frac{Z}{Z_0}$$

Normalized admittance :

$$y_n = \frac{1}{Z_n}$$

Drawing of Smith chart :

1. The Smith chart is a graphical means of obtaining line characteristics such as Γ, s and Z_{in}. It is constructed within a circle of unit radius ($|\Gamma| \le 1$) as shown in Fig. 5.27.1.

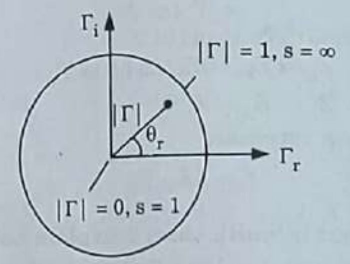

Fig. 5.27.1. Unit circle on which the Smith chart is constructed.

2. The construction of chart is based on a relation as

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad ...(5.27.1)$$

or

$$\Gamma = |\Gamma| \angle \theta_r = \Gamma_r + j\,\Gamma_i \quad ...(5.27.2)$$

where Γ_r and Γ_i are real and imaginary part of reflection coefficient.

3. The normalized impedance z_L is given by

$$z_L = \frac{Z_L}{Z_0} = r + jx \quad ...(5.27.3)$$

4. Substituting eq. (5.27.3) into eq. (5.27.1) and (5.27.2) gives

$$\Gamma = \Gamma_r + j\Gamma_i = \frac{z_L - 1}{z_L + 1} \quad ...(5.27.4)$$

or

$$z_L = r + j\,x = \frac{(1+\Gamma_r) + j\,\Gamma_i}{(1-\Gamma_r) - j\,\Gamma_i} \quad ...(5.27.5)$$

5. Equating real and imaginary components of eq. (5.27.5), we get

$$r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2} \quad ...(5.27.6)$$

$$x = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2} \quad ...(5.27.7)$$

6. Rearranging terms in eq. (5.27.6) and (5.27.7) leads to

$$\left[\Gamma_r - \frac{r}{1+r}\right]^2 + \Gamma_i^2 = \left[\frac{1}{1+r}\right]^2 \quad ...(5.27.8)$$

$$\text{and} \left[\Gamma_r - 1\right]^2 + \left[\Gamma_i - \frac{1}{x}\right]^2 = \left[\frac{1}{x}\right]^2 \quad ...(5.27.9)$$

Here centre at $(\Gamma_r, \Gamma_i) = \left(\frac{r}{r+1}, 0\right)$

$$\text{Radius} = \frac{1}{1+r}$$

7. For each r and x, there are two explicit circles (the resistance and reactance circles) and one implicit circle (the constant s-circle).

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Derive and explain differential form of Faraday's law of electromagnetic induction in vector form.

Ans. Refer Q. 5.1.

Q. 2. Explain the concept of displacement current in an electrical circuit. Also determine the condition when conduction current becomes equal to displacement current.

Ans. Refer Q. 5.2.

Q. 3. Explain all forms of Maxwell's equations in time varying conditions with its physical significance.

Ans. Refer Q. 5.6.

Q. 4. What do you mean by lossy dielectric and explain the wave propagation in lossy dielectric.

Ans. Refer Q. 5.7.

Q. 5. Discuss the propagation of plane waves in
i. Lossless dielectrics
ii. Free space
iii. Good conductors.

Ans. Refer Q. 5.10.

Q. 6. Explain Skin effect. Derive the expression for α and β in a conducting medium.

Ans. Refer Q. 5.12.

Q. 7. Explain Poynting vector. Derive an expression of Poynting theorem for EM wave. Also explain the significance of each term of the expression.

Ans. Refer Q. 5.17.

Q. 8. Explain the reflection of plane wave for the normal incidence. Discuss about reflection and transmission coefficient for *E* and *H*.

Ans. Refer Q. 5.18.

Q. 9. Derive expressions of voltage and current in a transmission line.

Ans. Refer Q. 5.20.

Q. 10. Define characteristic impedance.

Ans. Refer Q. 5.21.

Q. 11. Derive an expression for characteristic impedance, input impedance of a transmission line.

Ans. Refer Q. 5.24.

Q. 12. What is Smith chart ? Explain how it is constructed.

Ans. Refer Q. 5.27.

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1 UNIT

Co-ordinate Systems (2 Marks Questions)

1.1. Define the terms scalar and vector.

Ans.

A. **Scalar :** The term scalar refers to a quantity which has only magnitude but not direction.
Example : Mass, density etc.

B. **Vector :** The term vector refers to a quantity which has both magnitude and direction in space.
Example : Force, velocity etc.

1.2. Define unit vector.

Ans. A unit vector $\hat{a}_A$ along $\vec{A}$ is defined as a vector whose magnitude is unity and its direction is along $\vec{A}$, *i.e.*,

$$\hat{a}_A = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

1.3. Find the unit vector of the vector $\vec{A} = (7\hat{a}_x + 2\hat{a}_y + 8\hat{a}_z)$.

AKTU 2021-22, Marks 02

Ans. Unit vector of the vector $\vec{A}$,

$$\hat{a}_A = \frac{\vec{A}}{|\vec{A}|}$$

$$= \frac{7\hat{a}_x + 2\hat{a}_y + 8\hat{a}_z}{\sqrt{(7)^2 + (2)^2 + (8)^2}}$$

$$= \frac{7\hat{a}_x + 2\hat{a}_y + 8\hat{a}_z}{10.81}$$

$$= 0.64\hat{a}_x + 0.18\hat{a}_y + 0.74\hat{a}_z$$

1.4. Find the value of $(3\hat{a}_x + 6\hat{a}_y) \times (2\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z)$, where × denotes cross product.

AKTU 2021-22, Marks 02

Ans. Let,

$$\vec{A} = 3\hat{a}_x + 6\hat{a}_y$$
$$\vec{B} = 2\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$$

Cross product of vector $\vec{A}$ and $\vec{B}$,

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 3 & 6 & 0 \\ 2 & 3 & 5 \end{vmatrix}$$
$$= 30\hat{a}_x - 15\hat{a}_y - 3\hat{a}_z$$

1.5. Transform the point $P(5, 3, 6)$ in cylindrical coordinate system.

AKTU 2017-18, Marks 02

Ans.

Given : Point $P(5, 3, 6)$
To Transform : In cylindrical coordinate.

1. At point $P : x = 5, y = 3, z = 6$
2. Here,
$$\rho = \sqrt{x^2 + y^2} = \sqrt{5^2 + 3^2} = 5.83$$
$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{3}{5}$$
$$z = 6$$
3. Hence, $P(5, 3, 6)_{\text{Cartesion}} = P(5.83, 30.96°, 6)_{\text{Cylindrical}}$

1.6. Convert the point (– 2, 6, 3) into spherical coordinate system.

AKTU 2018-19, Marks 02

Ans.

1. At point $P(-2, 6, 3) : x = -2, y = 6$ and $z = 3$
2. Here,
$$r = \sqrt{x^2 + y^2 + z^2} = 7$$
$$\theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z} = \tan^{-1}\frac{\sqrt{40}}{3} = 64.62°$$
$$\phi = \tan^{-1} y/x = -71.56°$$
3. Point in spherical system, $P(7, 64.62°, -71.56°)$

1.7. Find the gradient of the scalar field $f(x, y, z) = x^2 + y + z$ at a point $P(2, 0, 1)$.

AKTU 2018-19, Marks 02

Ans.

$$f(x, y, z) = x^2 + y + z$$
$$\nabla f = \frac{\partial f}{\partial x}\hat{a}_x + \frac{\partial f}{\partial y}\hat{a}_y + \frac{\partial f}{\partial z}\hat{a}_z$$
$$= 2x\,\hat{a}_x + 1\,\hat{a}_y + 1\,\hat{a}_z$$

At point $P(2, 0, 1)$ *i.e.*, $x = 2, y = 0$ and $z = 1$,

$$\nabla f = 4\,\hat{a}_x + \hat{a}_y + \hat{a}_z$$

1.8. Prove that curl $\vec{A} = 0$, if $\vec{A} = (yz\hat{a}_x + zx\hat{a}_y + xy\hat{a}_z)$.

AKTU 2021-22, Marks 02

Ans.

1. Given, $\vec{A} = yz\hat{a}_x + zx\hat{a}_y + xy\hat{a}_z$
2. Curl of vector $\vec{A}$,
$$\nabla \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix}$$
$$= \hat{a}_x\left[\frac{\partial}{\partial y}xy - \frac{\partial}{\partial z}zx\right] - \hat{a}_y\left[\frac{\partial}{\partial x}xy - \frac{\partial}{\partial z}yz\right] + \hat{a}_z\left[\frac{\partial}{\partial x}zx - \frac{\partial}{\partial y}yz\right]$$
$$= \hat{a}_x[x - x] - \hat{a}_y[y - y] + \hat{a}_z[z - z]$$
$$= \hat{a}_x[0] - \hat{a}_y[0] + \hat{a}_z[0]$$

So $\nabla \times \vec{A} = 0$

3. Hence proved that curl of vector $\vec{A} = 0$

1.9. Define divergence theorem for a vector.

Ans. The divergence theorem states that the volume integral of the divergence of a vector field $\vec{F}$ taken over any volume v is equal to the surface integral of $\vec{F}$ taken over the closed surface surrounding the volume v.

Mathematically, $\iiint_v \left(\nabla \cdot \vec{F}\right) dv = \iint_S \vec{F} \cdot d\vec{S}$

i.e.,
$$\oint_S \vec{F} \cdot d\vec{S} = \int_v \nabla \cdot \vec{F}\, dv$$

1.10. State Stoke's theorem.

Ans. Stoke's theorem states that the circulation of vector field $\vec{A}$ around a closed path L is equal to the surface integral of the curl of $\vec{A}$ over the open surface S bounded by L, provided A and $\nabla \times A$ are continuous on S.

i.e.,
$$\oint_L \vec{A} \cdot \vec{dl} = \int_S \left(\nabla \times \vec{A}\right) \cdot d\vec{S}$$

1.11. Explain the physical significance of divergence and curl.

Ans.

A. Physical significance of divergence : "Divergence of a vector function $\vec{F}$ at each point gives the rate per unit volume at which the physical entity is issuing from that point".

B. Physical significance of curl : The physical significance of a curl of a vector field is that the curl provides the maximum value of the circulation of the field per unit area and indicates the direction along which this maximum value occurs.

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2 UNIT

Electrostatic (2 Marks Questions)

2.1. State Coulomb's law.

Ans. Coulomb's law states that force (F) between the two point charges Q_1 and Q_2 is :

1. Directly proportional to the product Q_1Q_2 of the charges.
2. Inversely proportional to the square of the distance R between them along the line joining them *i.e.*,

$$F \propto \frac{Q_1 Q_2}{R^2}$$

2.2. Explain electric field intensity. **AKTU 2021-22, Marks 02**

Ans. The electric field intensity $\vec{E}$ is defined as the force per unit charge when placed in an electric field.

i.e.,
$$\vec{E} = \frac{\vec{F}}{Q}$$

2.3. State point form of Ohm's law and Gauss's law.

AKTU 2017-18, Marks 02

OR

Explain Gauss's law for electrostatics.

AKTU 2019-20, Marks 02

Ans.

A. Ohm's law : The potential difference V across the terminals of a resistor R, is directly proportional to the current I flowing through it.

$$V \propto I$$
$$V = IR$$

B. Gauss's law : Gauss's law states that the total electric flux (ψ) through any closed surface is equal to the total charge enclosed by that surface. ...(2.3.1)

i.e.,
$$\psi = Q_{enc} = \oint_S \vec{D} \cdot d\vec{S}$$

2.4. Explain relaxation time constant.

AKTU 2019-20, Marks 02

AKTU 2018-19, Marks 02

Ans. The relaxation time constant (τ) is defined as the time required by the charge density to decay to 36.8 % of its initial value.

Relaxation time constant $(\tau) = \dfrac{\epsilon}{\sigma}$ sec.

2.5. Prove that $\vec{E}$ = – grad V, where E is electric field intensity and V is electric potential.

AKTU 2021-22, Marks 02

Ans.

$$dV = -E_x dx - E_y dy - E_z dz \quad ...(2.5.1)$$

and also

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz \quad ...(2.5.2)$$

Comparing eq. (2.5.1) and (2.5.2)

We get,

$$E_x = \frac{-\partial V}{\partial x}, E_y = \frac{-\partial V}{\partial y}, E_z = \frac{-\partial V}{\partial z}$$

$$\therefore \quad \vec{E} = -\nabla V = -(\text{grad } V)$$

2.6. Explain the significance of continuity equation in a good conductor.

AKTU 2019-20, Marks 02

Ans. The current diverging from a small volume element must be equal to the rate of decrease of charge within the volume.

2.7. Narrate the concept of electric dipole moment.

AKTU 2021-22, Marks 02

Ans.

1. It is the product of the magnitude of charges and the separation between them. The dipole moment determines the strength of an electric dipole to produce the electric field.
2. Mathematically, the electric dipole moment is given by,

$$P = q \times 2a$$

2.8. Write the Poisson's and Laplace equation.

AKTU 2017-18, Marks 02

Ans. Poisson's and Laplace's equations are used to solve the boundary value problems.

For a homogeneous medium,

$$\nabla^2 V = \frac{-\rho_v}{\epsilon} \quad ...(2.8.1)$$

Eq. (2.8.1) is called Poisson's equation.

For a charge free region, *i.e.*, $\rho_v = 0$

$$\nabla^2 V = 0 \quad ...(2.8.2)$$

Eq. (2.8.2) is called Laplace's equation.

2.9. A copper wire carries a conduction current of 1 amp at 60 Hz. What is the displacement current in the wire ? Assume $\mu = \mu_0$, $\epsilon = \epsilon_0$ and $\sigma = 5.8 \times 10^7$ ohm/m.

Ans.

Given : $I_C = 1$ A, $f = 60$ Hz, $\mu = \mu_0$, $\epsilon = \epsilon_0$ and $\sigma = 5.8 \times 10^7$ ohm/m

To Find : I_d

We know that,

$$J_d = \omega\epsilon E \text{ and } J_C = \sigma E$$

$$\therefore \quad \frac{J_d}{J_C} = \frac{\omega\epsilon}{\sigma} = \frac{I_d}{I_C}$$

$$I_d = I_C \frac{\omega\epsilon}{\sigma} = \frac{1 \times 2\pi \times 60 \times 10^{-9}}{36\pi \times 5.8 \times 10^7}$$

$$= 0.575 \times 10^{-16} \text{ A} = 5.75 \times 10^{-17} \text{ A}$$

2.10. Find electric field density for infinite line charge using Gauss's law.

Ans. According to Gauss's law

$$Q_{enc} = \oint_S \vec{D} \cdot \vec{dS}$$

$$\rho_L \cdot l = D \cdot 2\pi\rho l$$

$$\vec{D} = \frac{\rho_L}{2\pi\rho}\hat{a}_\rho = \epsilon_0 \vec{E}$$

$$\therefore \quad \vec{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\hat{a}_\rho$$

2.11. What is continuity equation ?

Ans. Continuity equation is derived from the law of conservation of charge and essentially states that there can be no accumulation of charge at any point.

Continuity equation is given by

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} \quad \text{where, } \rho_v = \text{Volume charge density}$$

2.12. Why work done on a charge is zero when it is moved in close path ? **AKTU 2019-20, Marks 02**

Ans. This is because starting and terminating point is same for a closed path. Hence upper and lower limit of integration becomes same hence the work done becomes zero. Such an integral over a closed path is denoted as,

$$\oint_{\text{Closed path}} \bar{E}\cdot d\bar{L} = 0$$

2.13. Prove line integral of static electric field in a close path is zero. **AKTU 2018-19, Marks 02**

Ans.

1. As we know that the potential difference between point A and B is independent of path taken. Hence,

$$V_{BA} = -V_{AB}$$

i.e.,
$$V_{BA} + V_{AB} = \oint_L \vec{E}.d\vec{l} \qquad ...(2.13.1)$$

2. The eq. (2.13.1) shows that the line integral of $\vec{E}$ along a closed path must be zero as shown in Fig. 2.13.1.
3. It also implies that no net work is done in moving a charge along a closed path in an electrostatic field.

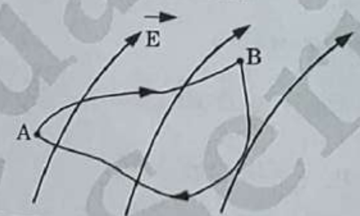

Fig. 2.13.1. The conservative nature of an electrostatic field.

4. Applying Stoke's theorem to eq. (2.13.1)

$$\oint_L \vec{E}.d\vec{l} = \int_S (\nabla \times \vec{E}).d\vec{S} = 0$$

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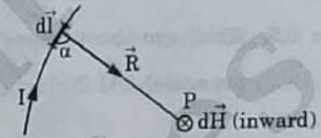
3 UNIT Magnetostatics (2 Marks Questions)

3.1. Explain Biot-Savart's Law. **AKTU 2017-18, Marks 02**

Ans. Biot-Savart's law states that the differential magnetic field intensity dH produced at a point P by the differential current element Idl is proportional to the product Idl and sine of angle (α) between the element and the line joining P to the element and is inversely proportional to the square of the distance R between P and the element. Mathematically,

$$dH \propto \frac{Idl \sin\alpha}{R^2}$$

or
$$dH = \frac{k\, Idl \sin\alpha}{R^2}$$



where k is the constant of proportionality.

Fig. 3.1.1. Magnetic field $\vec{dH}$ at P due to current element $\vec{Idl}$.

3.2. Name the two major laws governing magnetostatic field.

Ans. Two major laws are :
1. Biot-Savart's law.
2. Ampere's circuit law.

3.3. Explain Ampere's circuital law for magnetostatics. **AKTU 2019-20, Marks 02**

OR

Explain Ampere's circuital law in static magnetic field. **AKTU 2018-19, Marks 02**

Ans. Ampere's circuit law states that the line integral of magnetic field intensity around any closed path is exactly equal to the net current enclosed by that path.

Mathematically, $\oint \vec{H} \cdot d\vec{l} = I_{enc}$

3.4. Give Maxwell's equations in differential and integral form.

AKTU 2017-18, Marks 02

Ans.

Differential (or point) form	Integral Form	Remarks
$\nabla \cdot \vec{D} = \rho_v$	$\oint_S \vec{D} \cdot d\vec{S} = \int_v \rho_v \, dv$	Gauss's law
$\nabla \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot d\vec{S} = 0$	Non-existence of magnetic monopole
$\nabla \times \vec{E} = 0$	$\oint_L \vec{E} \cdot d\vec{l} = 0$	Conservative nature of electrostatic field
$\nabla \times \vec{H} = J$	$\oint_L \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{S}$	Ampere's law

3.5. What are the various characteristics of vector magnetic potential $(\vec{A})$?

Ans.

1. It exists even when $\vec{J}$ is present.
2. $\vec{A}$ is used to find near and far fields of antennas.

3.6. What are the characteristics of scalar magnetic potential (V_m) ?

Ans. 1. The negative gradient of V_m gives $\vec{H}$ *i.e.*, $\vec{H} = -\nabla V_m$

2. It exists where $\vec{J} = 0$.

3.7. The vector magnetic potential $\vec{A}$ due to a direct current in a conductor in free space is given by

$\vec{A} = (x^2 + y^2)\ \hat{a}_z\ \mu\text{Wb/m}^2$.

Determine the magnetic flux density $(\vec{B})$.

Ans. Given, $\vec{A} = (x^2 + y^2)\ \hat{a}_z\ \mu\text{Wb/m}^2$

$$\vec{B} = \nabla \times \vec{A}$$

$$= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & x^2 + y^2 \end{vmatrix} = \left[\hat{a}_x \left\{\frac{\partial}{\partial y}(x^2 + y^2)\right\} - \hat{a}_y \left\{\frac{\partial}{\partial x}(x^2 + y^2)\right\} + 0\right]$$

$$= [2y\ \hat{a}_x - 2x\ \hat{a}_y]\ \mu\text{Wb/m}^2.$$

3.8. Explain the concept of magnetic flux density.

AKTU 2021-22, Marks 02

Ans. The flux per unit Area (A), measured in a plane perpendicular to the flux is defined as the flux density. It is measured in Tesla (T) and denoted by B.

$\therefore$ Magnetic flux density $B = \dfrac{\phi}{A}$ Tesla.

3.9. Write the unit of magnetic flux density.

Ans. The unit of magnetic flux density is Wb/m^2.

3.10. An isotropic material has magnetic susceptibility of 4 and $\vec{H} = 1.989 \times 10^3 y\ \hat{a}_x$ A/m. Calculate magnetization $(\vec{M})$.

Ans. Given, $\chi_e = 4$, $\vec{H} = 1.989 \times 10^3\, y\, \hat{a}_x$ A/m

$$\vec{M} = \chi_e\ \vec{H} = 4 \times 1.989 \times 10^3\, y\, \hat{a}_x = 7.95 \times 10^3\, y\ \hat{a}_x \text{ A/m}$$

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4 UNIT

Magnetic Forces (2 Marks Questions)

4.1. Enlist the ways in which force due to magnetic fields can be experienced.

Ans.

1. Due to a moving charged particle in $\vec{B}$ field.
2. On a current element in an external magnetic field.
3. Between two current elements.

4.2. What is Lorentz force equation ?

Ans. For a moving charge Q in presence of both electric and magnetic field, total force on the charge is given by

$$\vec{F} = \vec{F}_e + \vec{F}_m$$

or

$$\vec{F} = Q(\vec{E} + \vec{u} \times \vec{B})$$

This is known as Lorentz force equation.

4.3. What is magnetic torque ?

Ans. The magnetic torque $\vec{T}$ on the loop is the vector product of the force $\vec{F}$ and the moment arm $\vec{r}$, *i.e.*, $\vec{T} = \vec{r} \times \vec{F}$

4.4. Write difference between magnetic and electric dipole.

Ans.

S. No.	Electric dipole	Magnetic dipole
1.	It is used to measure the separation of positive and negative charges in a system.	It is the torque experienced by a material placed in magnetic field.
2.	Materials made out of electric dipole cause the electric field that turns the dipole to be reduced.	Materials made out of magnetic dipole cause the magnetic field that turns the dipole to be increased.

4.5. Define magnetic dipole moment ?

Ans. It is the product of current and area of the loop. The direction of magnetic dipole moment is perpendicular to the loop and is given by

$$\vec{m} = IS\,\hat{a}_n$$

where, I = Current, S = Area of loop,

and $\hat{a}_n$ = Unit normal vector to the plane of loop.

4.6. What is magnetization ?

Ans. It is defined as the magnetic dipole moment per unit volume and is measured in amperes per meter. The total magnetization is given by

$$\vec{M} = \lim_{\Delta v \to 0} \frac{1}{\Delta v} \sum_{j=1}^{j=N} \vec{m}_j$$

where, m_j = Magnetic moment of j^{th} atom, Δv = Volume.

and N = Number of atoms.

4.7. Derive the expression for inductance per unit length of coaxial conductors.

Ans. The inductance per unit length is given by,

$$L = \frac{\phi}{I} = \frac{(\mu I / 2\pi)\log_e(a/b)}{I} = \frac{\mu}{2\pi}\log_e\left(\frac{a}{b}\right) \text{ H/m}$$

where, ϕ = Magnetic flux

a = Outer diameter of the conductor

b = Inner diameter of the conductor.

4.8. List any four analogies between electric and magnetic circuits.

Ans.

S. No.	Electric circuit	Magnetic circuit
1.	Current, $I = \int \vec{J} \cdot d\vec{S}$	Magnetic flux, $\psi = \int \vec{B} \cdot d\vec{s}$
2.	Current density, $\vec{J} = \frac{\vec{I}}{S} = \sigma \vec{E}$	Flux density, $\vec{B} = \frac{\psi}{S} = \vec{H}$

4.9. Explain the term 'Inductance'.

AKTU 2021-22, Marks 02

Ans. An inductor is a passive component that stores energy in the form of magnetic field and is also called as inductance.

4.10. Write the expression of energy stored in magnetic field.

Ans. Energy stored in magnetic field,

$$W = (1/2)LI^2$$

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UNIT 5 Waves and Applications (2 Marks Questions)

5.1. Explain Faraday's law. **AKTU 2017-18, Marks 02**

Ans. Faraday discovered that the induced emf, V_{emf} (in volts) in any closed circuit is equal to the time rate of change of the magnetic flux linkage by the circuit. This is called Faraday's law, and it can be expressed as

$$V_{emf} = \frac{-d\lambda}{dt} = -N\frac{d\psi}{dt}$$

where $\lambda = N\psi$ is the flux linkage, N is the number of turns in the circuit and Ψ is the flux through each turn.

5.2. Define the term depth of penetration.

Ans. The depth of penetration is defined as the depth at which waves attenuates to $1/e$ of its original amplitude. Depth of penetration is also called skin depth. It is a measure of depth to which an EM wave can penetrate the medium. Skin depth is given by,

$$\delta = \frac{1}{\alpha} \quad \text{where, } \alpha = \text{Attenuation constant.}$$

5.3. Explain behaviour of a conductor at high frequency. **AKTU 2019-20, Marks 02**

Ans. At high frequencies the skin depth becomes much smaller in conductor.

5.4. Explain Poynting vector. **AKTU 2019-20, Marks 02**

Ans. The vector product of electric field intensity $\vec{E}$ and magnetic field intensity $\vec{H}$ at any point is a measure of rate of energy flow per unit area at that point. The quantity $\vec{E} \times \vec{H}$ is known as the Poynting vector and is denoted by $\vec{P}$.

$$\vec{P} = \vec{E} \times \vec{H} \text{ W/m}^2$$

5.5. State Poynting's theorem.

Ans. Poynting's theorem states that the net power flowing out of a given volume is equal to the time rate of decrease in the energy stored within the volume minus the Ohmic losses.

Mathematically,

$$\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{S} = \frac{-\partial}{\partial t}\oint_v \left[\frac{1}{2}\in E^2 + \frac{1}{2}\mu H^2\right] dv - \oint_v \sigma E^2\, dv$$

Total power leaving the volume = Rate of decrease in energy stored in eletric and magnetic fields – Ohmic power dissipated

5.6. Explain reflection and transmission coefficients. **AKTU 2018-19, Marks 02**

Ans.

A. Reflection coefficient : It is the ratio of the magnitude of reflected electric field to incident electric field.

i.e., $$\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

B. Transmission coefficient : It is the ratio of the magnitude of transmitted electric field to incident electric field.

i.e., $$\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

5.7. Explain the significance of loss tangent.

Ans. tan θ or θ may be used to determine how lossy a medium is. A medium is said to be a good dielectric (lossless or perfect) if tan θ is very small ($\sigma << \omega\in$) or a good conductor if tan θ is very large ($\sigma >> \omega\in$).

5.8. Write an equation of EM wave. **AKTU 2018-19, Marks 02**

Ans. The wave equation is of the form

$$\frac{\partial^2\psi}{\partial t^2} - u^2\frac{\partial^2\psi}{\partial z^2} = 0$$

with the solution, $\psi = A \sin(\omega t - \beta z)$

5.9. Explain refraction coefficient and reflection time constant in context to EM wave propagation. **AKTU 2019-20, Marks 02**

Ans.

A. Refraction coefficient : The refraction coefficient represents the ratio of E (or H) in refracted and incident waves.

B. Reflection time constant (Reflection coefficient) : Refer Q. 5.6, Page SQ–16B, Unit-5, 2 Marks Questions.

5.10. Explain parameters of a transmission line.

AKTU 2019-20, Marks 02

Ans.

1. An electric transmission line has four parameters, namely resistance, inductance, capacitance and shunt conductance.
2. The electrical design and performance of a line are dependent on these parameters.
3. The line resistance and inductance form the series impedance of the line.
4. The capacitance and conductance form the shunt admittance of the line.

5.11. Give application of Smith chart. **AKTU 2017-18, Marks 02**

Ans.

1. Admittance calculations on any transmission line, on any load.
2. Impedance calculations on any transmission line, on any load.

5.12. Explain the physical significance of Poynting vector.

AKTU 2021-22, Marks 02

Ans. The integration of Poynting vector over any closed surface gives the net power flowing out of that surface.

5.13. Write an equation for an EM wave propagating in a conductor. **AKTU 2019-20, Marks 02**

Ans. The wave equations in a conductor,

$$\nabla^2 \vec{H} - \mu \left\{ \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} + \sigma \frac{\partial \vec{H}}{\partial t} \right\} = 0$$

5.14. Explain the reflection of a plane wave in a normal incidence.

AKTU 2021-22, Marks 02

Ans. If load impedance (Z_L) is not equal to characteristic impedance (Z_0) then mismatch occurs. In this case the part of the wave gets absorbed by the load while the rest part is reflected back to the source. Thus, due to mismatch in impedance the reflection occurs.

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B.Tech.

(SEM. IV) EVEN SEMESTER THEORY EXAMINATION, 2017-18

ELECTROMAGNETIC FIELD THEORY

Time : 3 Hours **Max. Marks : 100**

Note : Be precise in your answer. In case of numerical problem assume data wherever not provided.

SECTION – A

1. Attempt **all** parts of the following questions : **(2 × 10 = 14)**

a. Write the Poisson's and Laplace equation.

Ans. Refer Q. 2.8, Page SQ–6B, Unit-2, 2 Marks Questions.

b. State point form of Ohm's law & Gauss's law.

Ans. Refer Q. 2.3, Page SQ–5B, Unit-2, 2 Marks Questions.

c. Explain Biot-Savart's Law.

Ans. Refer Q. 3.1, Page SQ–9B, Unit-3, 2 Marks Questions.

d. Give Maxwell's equations in differential and integral form.

Ans. Refer Q. 3.4, Page SQ–10B, Unit-3, 2 Marks Questions.

e. Give application of Smith chart.

Ans. Refer Q. 5.11, Page SQ–16B, Unit-5, 2 Marks Questions.

f. Transform the point $P(5, 3, 6)$ in cylindrical coordinate system.

Ans. Refer Q. 1.5, Page SQ–2B, Unit-1, 2 Marks Questions.

g. Explain Faraday's law.

Ans. Refer Q. 5.1, Page SQ–14B, Unit-5, 2 Marks Questions.

SECTION-B

2. Attempt any **three** parts of the following questions :

a. Calculate the capacitance formed by two back to back cones separated by infinitely small distance.

Ans. Refer Q. 2.29, Page 2–34B, Unit-2.

b. State and derive ampere circuital law. A single turn circle coil of 50 meters in diameter carries current 28×10^4 amp. Determine the magnetic field intensity H at a point on the

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